Chapter 9

Inferences Based on Two Samples: Confidence Intervals and Tests of Hypothesis

Identifying the Target Parameter

Parameter	Key Words or Phrases
$\mu_1 - \mu_2$	Mean difference; difference in averages
$p_1 - p_2$	Difference between proportions, percentage, fractions or rates

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Paired Samples

Intervention Studies

"Before" and "After" Experiments

"Paired" samples

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Comparing Two Population Means: Paired Difference Experiments

TABLE 9.3 Reading Test Scores for Eight Pairs of Slow Learners

Pair	New Method (1)	Standard Method (2)
1	77	72
2	74	68
3	82	76
4	73	68
5	87	84
6	69	68
7	66	61
8	80	76

Is the mean reading test score using the new method greater than the mean reading test score using the old method?

$$H_0: (\mu_1 - \mu_2) = 0 \qquad H_a: (\mu_1 - \mu_2) > 0$$

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Comparing Two Population Means: Paired Difference Experiments

Because the samples are not independent of each other, a new technique is used

A new variable, d, is created

Testing is on the new variable, d

TABLE 9.4 Differences in Reading Test Scores

Pair	New Method	Standard Method	Difference (New Method − Standard Method)
1	77	72	5
2	74	68	6
3	82	76	6
4	73	68	5
5	87	84	3
6	69	68	1
7	66	61	5
8	80	76	4

$$H_0: \mu_D = 0 \ \ (\mu_1 - \mu_2) = 0$$
$$H_a: \mu_D > 0 \ \ (\mu_1 - \mu_2) > 0$$

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Comparing Two Population Means: Paired Difference Experiments

Testing is now based on a one sample t-statistic

$$t = \frac{\bar{d} - 0}{s_d / \sqrt{n_d}}$$

where $\bar{d}$ = Sample mean difference

s_d = Sample standard deviation of differences

n_d = Number of differences (number of pairs)

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Comparing Two Population Means: Paired Difference Experiments

- This type of experiment (paired observations) is called a paired difference experiment
- Pairing removes differences between pairs (days in this case), focuses on differences within pairs (sales)
- Comparisons within groups is called blocking
- Paired difference experiment is a randomized block experiment

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Comparing Two Population Means: Paired Difference Experiments

Paired Difference Confidence Interval for $\mu_d=\mu_1 - \mu_2$

Large Sample $$\bar{d} \pm z_{\alpha/2}\frac{\sigma_d}{\sqrt{n_d}} \approx z_{\alpha/2}\frac{s_d}{\sqrt{n_d}}$$

Small Sample $$\bar{d} \pm t_{\alpha/2}\frac{s_d}{\sqrt{n_d}}$$

where $t_{\alpha/2}$ is based on (n_d-1) degrees of freedom

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Comparing Two Population Means: Paired Difference Experiments

Paired Difference Test of Hypothesis for $\mu_d=\mu_1 - \mu_2$, Large Sample

One-Tailed Test	**Two-Tailed Test**
$H_0:(\mu_d) = D_0$	$H_0:(\mu_d) = D_0$
$H_a:(\mu_d) < D_0$	$H_a:(\mu_d) \neq D_0$
[or $H_a:(\mu_d) > D_0$]	

Test Statistic

$$z = \frac{\bar{d} - D_0}{\sigma_d / \sqrt{n_d}} \approx \frac{\bar{d} - D_0}{s_d / \sqrt{n_d}}$$

***Rejection region**: $z < -z_\alpha$* [or $z > z_\alpha$ when $H_a:(\mu_d) > D_0$]

***Rejection region**: $|z| > z_{\alpha/2}$*

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Comparing Two Population Means: Paired Difference Experiments

Paired Difference Test of Hypothesis for $\mu_d=\mu_1 - \mu_2$, Small Sample

One-Tailed Test	**Two-Tailed Test**
$H_0:(\mu_d) = D_0$	$H_0:(\mu_d) = D_0$
$H_a:(\mu_d) < D_0$	$H_a:(\mu_d) \neq D_0$
[or $H_a:(\mu_d) > D_0$]	

Test Statistic

$$t = \frac{\bar{d} - D_0}{s_d / \sqrt{n_d}}$$

***Rejection region**: $t < -t_\alpha$* [or $t > t_\alpha$ when $H_a:(\mu_d) > D_0$]

***Rejection region**: $|t| > t_{\alpha/2}$*

where t_α and $t_{\alpha/2}$ are based on (n_d-1) degrees of freedom

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Comparing Two Population Means: Paired Difference Experiments

Conditions for Valid Large-Sample Inferences about μ_d

1. Random sample of differences selected
2. Sample size is large ($n_d \geq 30$)

Conditions for Valid Small-Sample Inferences about μ_d

1. Random sample of differences selected
2. Population of differences has a distribution that is approximately normal

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Two Samples. Comparing Proportions

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Comparing Two Population Proportions: Independent Sampling

Properties of the Sampling Distribution of $(\hat{p}_1 - \hat{p}_2)$

- Mean of Sampling distribution $(\hat{p}_1 - \hat{p}_2)$ is $(p_1 - p_2)$;
- $E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2$; $(\hat{p}_1 - \hat{p}_2)$ is an unbiased estimator of $(p_1 - p_2)$
- Standard deviation of sampling distribution of $(\hat{p}_1 - \hat{p}_2)$ is

$$\sigma_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

- If n_1 and n_2 are large, the sampling distribution of $(\hat{p}_1 - \hat{p}_2)$ is approximately normal

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Comparing Two Population Proportions: Independent Sampling

Large-Sample 100(1-α)% Confidence Interval for (p_1-p_2)

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sigma_{(\hat{p}_1 - \hat{p}_2)} = (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

$$\approx (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

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Comparing Two Population Proportions: Independent Sampling

Conditions required for Valid Large-Sample Inferences about (p_1-p_2)

- Independent, randomly selected samples
- Sample sizes n_1 and n_2 are sufficiently large so that the sampling distribution of $(\hat{p}_1 - \hat{p}_2)$ will be approximately normal.

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Comparing Two Population Proportions: Independent Sampling

Large-Sample Test of Hypothesis about (p_1-p_2)

One-Tailed Test	**Two-Tailed Test**
$H_0: (p_1-p_2) = 0$	$H_0: (p_1-p_2) = 0$
$H_a: (p_1-p_2) < 0$	$H_a: (p_1-p_2) \neq 0$
{or $H_a: (p_1-p_2) > 0$]	

Test Statistic

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sigma_{(\hat{p}_1 - \hat{p}_2)}}$$

Rejection region: $z < -z_\alpha$ [or $z > z_\alpha$ when $H_a: (\mu_1-\mu_2) > D_0$]	*Rejection region:* $/z/ > z_{\alpha/2}$

Note: $\sigma_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \approx \sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$, where $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$

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Alternative formula

$$\sigma = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

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