# Chapter 8

Inferences Based on a Single Sample: Tests of Hypothesis

## The Elements of a Test of Hypothesis

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1. The Null hypothesis
2. The alternate, or research hypothesis
3. The test statistic
4. The rejection region
5. The assumptions
6. The Experiment and test statistic calculation
7. The Conclusion

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## The Elements of a Test of Hypothesis

Does a manufacturer's pipe meet building code?

Null hypothesis – Pipe does not meet code

($H_0$): $\mu \leq 2400$

Alternate hypothesis – Pipe meets specifications

($H_a$): $\mu > 2400$

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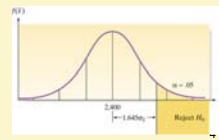
## The Elements of a Test of Hypothesis

Test statistic to be used $z = \frac{\bar{x} - 2400}{\sigma_{\bar{x}}} = \frac{\bar{x} - 2400}{\sigma / \sqrt{n}}$

Rejection region

Determined by Type I error, which is the probability of rejecting the null hypothesis when it is true, which is $\alpha$. Here, we set $\alpha$=.05

Region is z>1.645, from z value table



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## The Elements of a Test of Hypothesis

Assume that s is a good approximation of $\sigma$

Sample of 60 taken, $\bar{x} = 2460$ , s=200

Test statistic is $z = \frac{\bar{x} - 2400}{s / \sqrt{n}} = \frac{2460 - 2400}{200 / \sqrt{60}} = \frac{60}{28.28} = 2.12$

Test statistic lies in rejection region, therefore we reject $H_0$.

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## The Elements of a Test of Hypothesis

Type I vs Type II Error

**Conclusions and Consequences for a Test of Hypothesis**

| | True State of Nature | |
|---|---|---|
| Conclusion | $H_0$ True | $H_a$ True |
| Fail to reject $H_0$ | Correct decision | Type II error (probability $\beta$) |
| Reject $H_0$ | Type I error (probability $\alpha$) | Correct decision |

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## The Elements of a Test of Hypothesis

1. The Null hypothesis – the status quo. What we will accept unless proven otherwise. Stated as $H_0$: *parameter = value*
2. The Alternative (research) hypothesis *($H_a$)* – theory that contradicts $H_0$. Will be accepted if there is evidence to establish its truth
3. Test Statistic – sample statistic used to determine whether or not to reject *Ho* and accept $H_a$

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## The Elements of a Test of Hypothesis

4. The rejection region – the region that will lead to $H_0$ being rejected and $H_a$ accepted. Set to minimize the likelihood of a Type I error
5. The assumptions – clear statements about the population being sampled
6. The Experiment and test statistic calculation – performance of sampling and calculation of value of test statistic
7. The Conclusion – decision to (not) reject $H_0$, based on a comparison of test statistic to rejection region

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## Large-Sample Test of Hypothesis about a Population Mean

Null hypothesis is the status quo, expressed in one of three forms

$H_0$: $\mu = 2400$

$H_0$: $\mu \le 2400$

$H_0$: $\mu \ge 2400$

We can either "Reject" or "Fail to reject" the Null.

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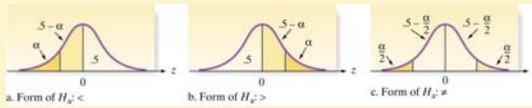
## Large-Sample Test of Hypothesis about a Population Mean

Alternative hypothesis can take one of 3 forms:

| | |
|---|---|
| One-tailed, lower tail | $H_a$: $\mu<2400$ |
| One-tailed, upper tail | $H_a$: $\mu>2400$ |
| Two-tailed | $H_a$: $\mu\neq2400$ |



a. Form of $H_a$: <   b. Form of $H_a$: >   c. Form of $H_a$: ≠

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## Large-Sample Test of Hypothesis about a Population Mean

**TABLE 8.2 Rejection Regions for Common Values of $\alpha$**

| | Alternative Hypotheses | | |
|---|---|---|---|
| | Lower-Tailed | Upper-Tailed | Two-Tailed |
| $\alpha = .10$ | $z < -1.28$ | $z > 1.28$ | $z < -1.645$ or $z > 1.645$ |
| $\alpha = .05$ | $z < -1.645$ | $z > 1.645$ | $z < -1.96$ or $z > 1.96$ |
| $\alpha = .01$ | $z < -2.33$ | $z > 2.33$ | $z < -2.575$ or $z > 2.575$ |

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## Large-Sample Test of Hypothesis about a Population Mean

If we have: n=100, $\bar{x}$ = 11.85, s = .5, and we want to test if $\mu \neq$ 12 with a 99% confidence level, our setup would be as follows:

$H_0$: $\mu = 12$

$H_a$: $\mu \neq 12$

Test statistic $z = \frac{\bar{x} - 12}{\sigma_{\bar{x}}}$

Rejection region z < -2.575 or z > 2.575 (two-tailed)

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## Large-Sample Test of Hypothesis about a Population Mean

CLT applies, therefore no assumptions about population are needed

Solve $z = \frac{\bar{x}-12}{\sigma_{\bar{x}}} = \frac{\bar{x}-12}{\sigma/\sqrt{n}} = \frac{11.85-12}{\sigma/\sqrt{100}} \approx \frac{11.85-12}{s/10} = \frac{-.15}{.5/10} = -.3$

Since z falls in the rejection region, we conclude that at .01 level of significance the observed mean differs significantly from 12

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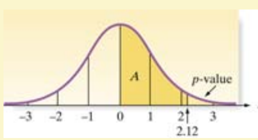
## Observed Significance Levels: p-Values

The p-value, or observed significance level, is the smallest level of α at which we can reject the Null.

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## Observed Significance Levels: p-Values

When p-values are used, results are reported by setting the maximum α you are willing to tolerate, and comparing p-value to that to reject or not reject $H_0$

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## Small-Sample Test of Hypothesis about a Population Mean

When sample size is small (<30) we use a different sampling distribution for determining the rejection region and we calculate a different test statistic

The t-statistic and t distribution are used in cases of a small sample test of hypothesis about μ

All steps of the test are the same, and an assumption about the population distribution is now necessary, since CLT does not apply

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## Small-Sample Test of Hypothesis about a Population Mean

**Small-Sample Test of Hypothesis about $\mu$**

| One-Tailed Test | Two-Tailed Test |
|---|---|
| $H_0$: $\mu = \mu_0$ | $H_0$: $\mu = \mu_0$ |
| $H_a$: $\mu < \mu_0$ (or $H_a$: $\mu > \mu_0$ ) | $H_a$: $\mu \neq \mu_0$ |
| Test Statistic: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ | Test Statistic: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ |
| Rejection region: $t < -t_\alpha$ (or $t > t_\alpha$ when $H_a$: $\mu > \mu_0$ | Rejection region: $\lvert t \rvert > t_{\alpha/2}$ |

where $t_\alpha$ and $t_{\alpha/2}$ are based on (n-1) degrees of freedom

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## Large-Sample Test of Hypothesis about a Population Proportion

**Large-Sample Test of Hypothesis about $p$**

| One-Tailed Test | Two-Tailed Test |
|---|---|
| $H_0$: $p = p_0$ | $H_0$: $p = p_0$ |
| $H_a$: $p < p_0$ (or $H_a$: $p > p_0$ ) | $H_a$: $p \neq p_0$ |
| Test Statistic: $z = \frac{\hat{p}-p_0}{\sigma_{\hat{p}}}$ | Test Statistic: $z = \frac{\hat{p}-p_0}{\sigma_p}$ |

where, according to $H_0$, $\sigma_{\hat{p}} = \sqrt{p_0 q_0/n}$ and $q_0 = 1 - p_0$

| | |
|---|---|
| Rejection region: $z < -z$ (or $z > z_\alpha$ when $p > p_0$ | Rejection region: $\lvert z \rvert > z_{\alpha/2}$ |

## Large-Sample Test of Hypothesis about a Population Proportion

Assumptions needed for a Valid Large-Sample Test of Hypothesis for p

- A random sample is selected from a binomial population
- The sample size n is large (condition satisfied if $p_0 \pm 3\sigma_{\hat{p}}$ falls between 0 and 1

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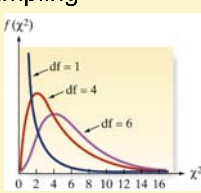
## Tests of Hypothesis about a Population Variance

Hypotheses about the variance use the Chi-Square distribution and statistic

The quantity $\frac{(n-1)s^2}{\sigma^2}$ has a sampling distribution that follows the chi-square distribution assuming the population the sample is drawn from is normally distributed.

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## Tests of Hypothesis about a Population Variance

**Small-Sample Test of Hypothesis about $\sigma^2$**

| One-Tailed Test | Two-Tailed Test |
|---|---|
| $H_0$: $\sigma^2 = \sigma_0^2$ | $H_0$ $\sigma^2 = \sigma_0^2$ |
| $H_a$: $\sigma^2 < \sigma_0^2$ or $H_a$: $\sigma^2 > \sigma_0^2$ | $H_a$ $\sigma^2 \neq \sigma_0^2$ |
| Test Statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$ | Test Statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$ |
| Rejection region: $\chi^2 < \chi^2_{(1-\alpha)}$ | Rejection region $\chi^2 < \chi^2_{(1-\alpha/2)}$ |
| (or $\chi^2 > \chi^2_{\alpha}$ when $H_a$: $\sigma^2 > \sigma_0^2$ | Or $\chi^2 > \chi^2_{(\alpha/2)}$ |

where $\sigma_0^2$ is the hypothesized variance and the distribution of $\chi^2$ is based on (n-1) degrees of freedom