Chapter 4

Discrete Random Variables

Two Types of Random Variables

Random Variable

- –variable that assumes numerical values associated with random outcomes of an experiment
- –Only one numerical value is assigned to each sample point

•Two types of Random Variable

- –Discrete
- –Continuous

Two Types of Random Variables

Discrete Random Variable

- –Random variable that has a finite, or countable number of distinct possible values
- –Example – number of people born in July

•Continuous Random Variable

- –Random variable that has an infinite number of distinct possible values
- –Average age of people born in July

Probability Distributions for Discrete Random Variables

2 Requirements that must be satisfied

1. $p(x) \geq 0$ for all values of x
2. $\sum p(x) = 1$ Where the summation of p(x) is over all possible values of x

Probability Distributions for Discrete Random Variables

Experiment - tossing 2 coins simultaneously
Random variable X – number of heads observed
X can assume values of 0, 1 and 2

Calculate the probability associated with each value of X

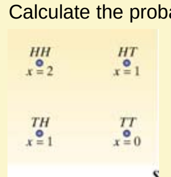


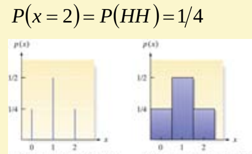
TABLE 4.1 Probability Distribution for Coin-Toss Experiment: Tabular Form

x	p(x)
0	1/4
1	1/2
2	1/4

Probability Distributions for Discrete Random Variables

Probability Distribution of Discrete Random Variable X – Other forms

$$P(x=0) = P(TT) = 1/4$$
$$P(x=1) = P(TH) + P(HT) = 1/4 + 1/4 = 1/2$$
$$P(x=2) = P(HH) = 1/4$$



Expected Values of Discrete Random Variables

The mean, or expected value of a discrete random variable is:

$$\mu = \mathrm{E}(x) = \sum xp(x)$$

Expected Value of x (number of heads observed)		
X	P(x)	Xp(x)
0	¼	0
1	½	½
2	¼	½
Expected Value		1

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Expected Values of Discrete Random Variables

The variance of a discrete random variable is:

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum (x-\mu)^2 p(x)$$

$$= (0-1)^2 \bullet \frac{1}{4} + (1-1)^2 \bullet \frac{1}{2} + (2-1)^2 \bullet \frac{1}{4} = \frac{1}{2}$$

and standard deviation is

$$\sigma = \sqrt{\sigma^2} = \sqrt{1/2}$$

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Expected Values of Discrete Random Variables

Probability Rules for a Discrete Random Variable

Probability Rules for a Discrete Random Variable		
	Chebyshev's Rule	Empirical Rule
	Applies to any distribution	Applies to mound-shaped and symmetric distributions
$P(\mu-\sigma < x < \mu+\sigma)$	≥ 0	$\approx .68$
$P(\mu-2\sigma < x < \mu+2\sigma)$	$\geq 3/4$	$\approx .95$
$P(\mu-3\sigma < x < \mu+3\sigma)$	$\geq 8/9$	≈ 0.997

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The Binomial Random Variable

Binomial Random variable

- An experiment of n identical trials
- 2 possible outcomes on each trial, denoted as S(success) and F(failure)
- Probability of success (p) is constant from trial to trial. Probability of failure (q) is 1-p
- Trials are independent
- Binomial random variable – number of S's in n trials

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The Binomial Random Variable

Heart association claims that only 10% of US adults over 30 can pass the President's Physical Fitness commission's minimum requirements.

Select 4 adults at random, administer the test. What is the probability that none of the adults passes the test?

TABLE 4.2 Sample Points for Fitness Test of Example 4.9

SSSS	*FSSS*	*FFSS*	*SFFF*	*FFFF*
	SFSS	*FSFS*	*FSFF*	
	SSFS	*FSSF*	*FFSF*	
	SSSF	*SFFS*	*FFFS*	
		SFSF		
		SSFF		

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S = pass, F = Fail

The Binomial Random Variable

Use multiplicative rule to calculate probabilities of the possible outcomes

P(SSSS) = .1*.1*.1*.1=.1^4=.0001

P(FSSS) = .9*.1*.1*.1=.9*.1^3=.0009

…..

P(FFFF) = .9*.9*.9*.9=.9$^{4=}$.6561

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The Binomial Random Variable

What is the probability that 3 of the 4 adults pass the test?

P(3 of the 4 adults pass the test) = $4(.1)^3(.9)=4(.09) = .0036$

What is the probability that 3 of the 4 adults fail the test?

P(3 of the 4 adults fail the test) = $4(.9)^3(.1)=4(.0729) = .2916$

Do you see a pattern?

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The Binomial Random Variable

Formula for the probability distribution p(x)

$$p(x) = \binom{n}{x} \bullet p^x q^{n-x}$$

Where p = probability of success on single trial

q = 1-p

n = Number of trials

x = number of successes in n trials

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

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The Binomial Random Variable

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard deviation $\sigma = \sqrt{npq}$

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The Binomial Random Variable

Using Binomial Tables

Binomial tables are cumulative tables, entries represent cumulative binomial probabilities

Make use of additive and complementary properties to calculate probabilities of individual x's, or x being greater than a particular value.

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The Binomial Random Variable

If $x \leq 2$, and p =.1, n =10, then $P(x \leq 2)$ =.930

If x = 2, and p =.1, n =10, then P(x=2) = $P(x \leq 2) - P(x \leq 1)$=.930-.736 = .194

If x >2, and p = .1, n =10, then P(x>2) = 1- $P(x \leq 2)$ =1-.930 = .322

TABLE 4.4 Reproduction of Part of Table II of Appendix A: Binomial Probabilities for $n = 10$

k \ p	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99
0	.904	.599	.349	.107	.028	.006	.001	.000	.000	.000	.000	.000	.000
1	.996	.914	.736	.376	.149	.046	.011	.002	.000	.000	.000	.000	.000
2	1.000	.988	.930	.678	.383	.167	.055	.012	.002	.000	.000	.000	.000
3	1.000	.999	.987	.879	.650	.382	.172	.055	.011	.001	.000	.000	.000
4	1.000	1.000	.998	.967	.850	.633	.377	.166	.047	.006	.000	.000	.000
5	1.000	1.000	1.000	.994	.953	.834	.623	.367	.150	.033	.002	.000	.000
6	1.000	1.000	1.000	.999	.989	.945	.828	.618	.350	.121	.013	.001	.000
7	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.322	.070	.012	.000
8	1.000	1.000	1.000	1.000	1.000	.988	.989	.954	.851	.624	.264	.086	.004
9	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.893	.651	.401	.096

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