Chapter 3

Probability

Events, Sample Spaces and Probability

What do you do when the number of sample points is too large to enumerate?

Use the Combinations Rule to count number of samples possible when selecting sample of size n from N elements

$$\binom{N}{n} = \frac{N!}{n!(N-n!)}$$

where

$$n! = n(n-1)(n-2)\ldots(3)(2)(1)$$

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Events, Sample Spaces and Probability

In the dice throwing example, there are 36 pairings. How many different samples of 2 pairs can we select from those 36 pairs?

$$\binom{36}{2} = \frac{36!}{2!(36-2)!} = \frac{36!}{2!34!} = \frac{36\times 35\times 34\times\cdots\times 3\times 2\times 1}{2\times 1(34\times 33\times\cdots\times 2\times 1)} = \frac{1260}{2} = 630$$

We have 630 possible samples of 2 pairs from a group of 36 pairs

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Events, Sample Spaces and Probability

If you had 30 people interested in being in a study and you needed 5, how many different combinations of 5 are there?

$$\binom{30}{5} = \frac{30!}{5!(30-5)!} = \frac{30!}{5!25!} = \frac{30\bullet 29\bullet 28\bullet 27\bullet 26\bullet 25....3\bullet 2\bullet 1}{5\bullet 4\bullet 3\bullet 2\bullet 1(25\bullet 24...3\bullet 2\bullet 1)} = 142{,}506$$

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Random Sampling

Assume a desired sample size of n

Sample is random if every set of n elements in the population has the same probability of being selected

Random number generators often used to produce a random sample

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Some Additional Counting Rules

The Multiplicative Rule – With k sets of n_1, n_2,…n_k elements, the number of different samples that can be formed by taking one element from each of the k sets is

$$n_1 n_2 n_3 \ldots n_k$$

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Some Additional Counting Rules

The Permutations Rule – With a single set of N distinct elements, the number of different samples that can be formed by taking n elements at a time and arranging them within n positions is

$$P_n^N = N(N-1)(N-2)\cdots(N-n+1) = \frac{N!}{(N-n)!}$$

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Some Additional Counting Rules

The Partitions Rule – With a single set of N distinct elements, the number of different ways to partition the elements into k sets of n_1, n_2,…n_k elements is

$$\frac{N!}{n_1!n_2!\cdots n_k!} \quad where\ n_1 + n_2 + n_3 + \cdots + n_k = N$$

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