Chapter 3

Probability

Objectives

- Develop probability as a measure of uncertainty
- Introduce basic rules for finding probabilities
- Use probability as a measure of reliability for an inference

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Events, Sample Spaces and Probability

Experiment – process of observation that leads to a single outcome with no predictive certainty

Sample point – most basic outcome of an experiment

Sample Space – a listing of all sample points for an experiment

Experiment – tossing 2 coins

A Sample Point – HT

Sample Space – S: {HH, HT, TH, TT}

Sample point probability – relative frequency of the occurrence of the sample point

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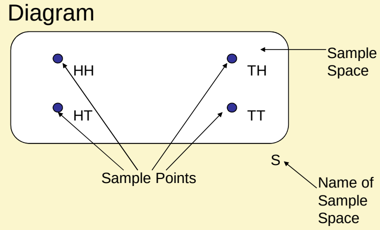
Events, Sample Spaces and Probability

Venn Diagram



Sample Point Probabilities must lie between 0 and 1

The sum of all sample point probabilities must be one

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Events, Sample Spaces and Probability

How to assign Sample Point Probabilities?

- –Prior knowledge/assumption
- –Multiple repetitions of an experiment
- –Estimation based on survey

Event – a specific collection of sample points

Probability of an event – the sum of the probabilities of all sample points in the collection for the event

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Events, Sample Spaces and Probability

How to assign Event Probabilities?

- –Define experiment
- –List sample points
- –Assign probabilities to sample points
- –Identify collection of sample points in Event
- –Sum sample point probabilities

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Events, Sample Spaces and Probability

What is the probability of rolling an eight in a single toss of a pair of dice?

Experiment is toss of pair of dice

1,1 1/36	1,2 1/36	1,3 1/36	1,4 1/36	1,5 1/36	1,6 1/36
2,1 1/36	2,2 1/36	2,3 1/36	2,4 1/36	2,5 1/36	2,6 1/36
3,1 1/36	3,2 1/36	3,3 1/36	3,4 1/36	3,5 1/36	3,6 1/36
4,1 1/36	4,2 1/36	4,3 1/36	4,4 1/36	4,5 1/36	4,6 1/36
5,1 1/36	5,2 1/36	5,3 1/36	5,4 1/36	5,5 1/36	5,6 1/36
6,1 1/36	6,2 1/36	6,3 1/36	6,4 1/36	6,5 1/36	6,6 1/36

Probability of rolling an 8 = 1/36+1/36+1/36+1/36+1/36 = 5/36= **.14**

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Events, Sample Spaces and Probability

What is the probability of rolling at least a 9 with a single toss of two dice?

P(at least 9) = P(9) + P(10) + P(11) + P(12)

= 4/36 + 3/36 + 2/36 + 1/36

= 10/36

= 5/18 = **.28**

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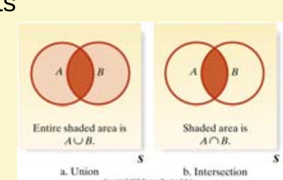
Unions and Intersections

Compound Event – a composition of 2 or more events

Can be the result of a union or intersection of events



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Unions and Intersections

Event A – A New Jersey Birth Mother is white

Event B – A New Jersey Birth Mother was a teenager when giving birth

$P(A) = .79$

$P(B) = .09$

$P(A \cap B) = .05$

TABLE 3.4 Percentage of New Jersey Birth Mothers in Age–Race Classes

Maternal Age (years)	Race: White	Race: Black
≤17	2%	2%
18–19	3%	2%
20–29	41%	12%
≥30	33%	5%

Source: Reichman, N. E., and Pagnini, D. L. "Maternal age and birth outcomes: Data from New Jersey." *Family Planning Perspectives*, Vol. 29, No. 6, Nov./Dec. 1997, p. 269 (adapted from Table 1).

$P(A \cup B) = .02 + .03 + .41 + .33 + .02 + .02 = .83$

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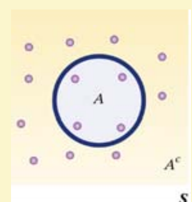
Complementary Events

Complementary Event – The complement of Event A, A^C is all sample points that do not belong to Event A



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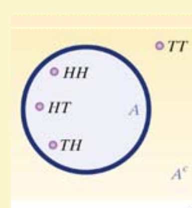
Complementary Events

If A is having at least 1 head appear in the toss of 2 coins, A^C is having no heads appear



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The Additive Rule and Mutually Exclusive Events

The Additive Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) =$$

$$.79 + .09 - .05 = .83$$

TABLE 3.4 Percentage of New Jersey Birth Mothers in Age–Race Classes

	Race	
Maternal Age (years)	White	Black
≤17	2%	2%
18–19	3%	2%
20–29	41%	12%
≥30	33%	5%

Source: Reichman, N. E., and Pagnini, D. L. "Maternal age and birth outcomes: Data from New Jersey." *Family Planning Perspectives*, Vol. 29, No. 6, Nov./Dec. 1997, p. 269 (adapted from Table 1).

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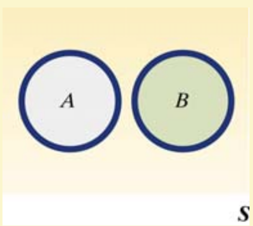
The Additive Rule and Mutually Exclusive Events

Mutually Exclusive Events – Events are mutually exclusive if they share no sample points.



$$P(A \cap B) = 0$$

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The Additive Rule and Mutually Exclusive Events

The Additive Rule for Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$

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Conditional Probability

Conditional Probability – the probability that event A occurs given that event B occurs

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional probability works with a reduced sample space, the space that contains B and $A \cap B$

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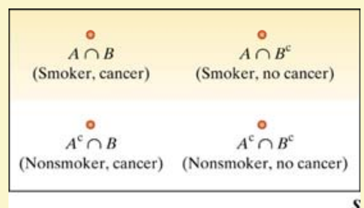
Conditional Probability

Sample space for $A|B$



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Conditional Probability

Event A – cause of complaint is appearance

Event B – complaint occurred during guarantee period

TABLE 3.6 Distribution of Product Complaints

	Reason for Complaint			
	Electrical	Mechanical	Appearance	Totals
During Guarantee Period	18%	13%	32%	63%
After Guarantee Period	12%	22%	3%	37%
Totals	30%	35%	35%	100%

$P(A \cap B) = .32$ $\qquad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.32}{.63} = .51$

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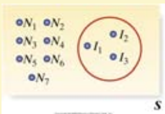
The Multiplicative Rule and Independent Events

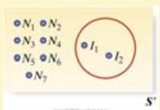
The Multiplicative Rule

$$P(A \cap B) = P(A)P(B|A) \quad \text{or} \quad P(A \cap B) = P(B)P(A|B)$$



$$A = \{I_1, I_2, I_3\}$$

$$P(A) = P(I_1) + P(I_2) + P(I_3) = \tfrac{1}{10} + \tfrac{1}{10} + \tfrac{1}{10} = \tfrac{3}{10}$$



$$B|A = \{I_1, I_2\}$$

$$P(B|A) = P(I_1) + P(I_2) = \tfrac{1}{9} + \tfrac{1}{9} = \tfrac{2}{9}$$

$$P(A \cap B) = P(A) \bullet P(B|A) = \tfrac{3}{10} \bullet \tfrac{2}{9} = \tfrac{6}{90} = \tfrac{1}{15}$$

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The Multiplicative Rule and Independent Events

Events A and B are independent if the occurrence of one does not alter the probability of the other occurring

$$P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)$$

If A and B are independent events

$$P(A \cap B) = P(A) \bullet P(B)$$

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The Multiplicative Rule and Independent Events

Event A – cause of complaint is appearance

Event B – complaint occurred during guarantee period

Distribution of Product Complaints

	Reason for Complaint			
Complaint Origin	Electrical	Mechanical	Appearance	Totals
During Guarantee Period	18%	13%	32%	63%
After Guarantee Period	12%	22%	3%	37%
Totals	30%	35%	35%	100%

Are A and B independent events?

$$P(A|B) = .51 \qquad P(A) = .32 + .03 = .35$$

A and B are not independent

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Bayes's Rule

Allows calculation of unknown conditional probability from known conditional probability

$$P(B_i|A) = P(B_i \cap A)/P(A) = \frac{P(B_i)P(A|B_i)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \cdots + P(B_k)P(A|B_k)}$$

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