Statistics 515 Fall 2007 Exam 2.
You may use the formula sheet, Z, t and Chi-square tables and a calculator.

Note. For all tests of hypothesis you have to define clearly the null and alternative hypothesis, the test statistics, critical region, and conclusion.
"CI" stands for "Confidence Interval".

Please, answer the following questions:
Problem 1 (20 points)
A sample of size 25 results in mean of 10 and standard deviation of 2.

a) Construct 95% CI for the population mean.

$$\overline{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 10 \pm 2.064\frac{2}{\sqrt{25}} = 10 \pm 0.8256;$$

$$95\%\ CI\ \ (9.1744, 10.8256)$$

b) What assumptions did you have to make in part **a)** above?
Normal population, random sample

c) Explain the meaning of the expression:
"We are 95% confident that the true population mean is included in this confidence interval"
If we draw many large samples on average 95% of the CIs will contain the real population parameter. With only one sample our confidence is related to the estimation process not to the sample data.

Problem 2 (20 points).
A sample of size 31 results in mean of 20 and standard deviation of 1.
a) Construct 95% CI for the population variance.

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right) = \left(\frac{(31-1)1^2}{46.9792}, \frac{(31-1)1^2}{16.7908}\right)$$

$$95\%\ CI\ \ (0.639, 1.787)$$

b) Test the hypothesis that the population variance is equal to 2 versus the hypothesis that the population variance is not equal to 2 at $\alpha = .05$.

$H_0:\ \sigma^2 = 2$

$H_a:\ \sigma^2 \neq 2$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(31-1)1^2}{2} = 15$$

Critical region $\chi^2_{.975} = 16.7908$ and $\chi^2_{.025} = 46.9792$

$15 < 16.7908 \ \rightarrow$ Reject H_0.

c) If you **increase** the sample size what happens to the width of CI? **Decreases.**

d) If you **decrease** the confidence from 95% to 90% what happens to the width of CI? **Decreases.**

Problem 3 (20 points).
A sample of size 16 results in mean of 5 and standard deviation of 2.
a) Test the hypothesis that the population mean is equal to 4 versus the hypothesis that it is not equal to 4 at $\alpha = .01$.

$H_0 : \mu = 4$

$H_a : \mu \neq 4$

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}} = \frac{5-4}{2/\sqrt{16}} = 2$$

Critical Region $t_{\substack{.005 \\ df=15}} = 2.947$

$2 < 2.947 \rightarrow \textit{Fail to reject } H_0$

b) Test the hypothesis that that the population mean is equal to 4 versus the hypothesis that it is greater than 4 at $\alpha = .05$.

$H_0 : \mu = 4$

$H_a : \mu > 4$

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}} = \frac{5-4}{2/\sqrt{16}} = 2$$

Critical Region $t_{\substack{.05 \\ df=15}} = 1.753$

$2 > 1.753 \rightarrow \textit{Reject } H_0$

c) What assumptions did you have to make for part a) and b) above?
Random samples and normal populations.

Problem 4 (20 points).
From a random sample of 100 newborn babies, 20 of them have birth weight above the norm.

a) Is this a large sample for the purposes of testing for population proportion? Explain.
Large sample because $np \geq 5$ and $nq \geq 5$; $p = 20/100 = .2$;100*.2=20 ≥ 5 and 100*.8=80 ≥ 5

b) Test the hypothesis at α=.1 that the population proportion of newborn babies with birth weight above the norm is less than 0.3.

$H_0 : p = 0.3$

$H_a : p < 0.3$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.2 - .3}{\sqrt{\frac{.3 * .7}{100}}} = -2.18$$

$z_{.1} = -1.28$

$-2.18 < -1.28$ Reject H_0

c) Define "Type I error"
Reject H_0 when it is true.

d) Define "Type II error"
Fail to reject H_0 when H_a is true.

Problem 5 (20 points).

Sample one: size of 100, proportion $p_1 = 0.4$; Sample two: size of 200, proportion $p_2 = 0.3$.

a) Are these two samples large or small for the purposes of testing for difference in population proportions? Explain.

Both are large samples. Check: $np \geq 5$ and $nq \geq 5$ *or* $\hat{p} \pm 3\sigma_{\hat{p}}$ does not include 0 or 1.

b) Construct 95% CI for the difference $(p_1 - p_2)$.

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} = (0.4 - 0.3) \pm 1.96\sqrt{\frac{0.4*0.6}{100} + \frac{0.3*0.7}{200}} =$$

$$= 0.1 \pm 0.115$$

$$95\% CI \ (-0.015, 0.215)$$

c) Test the hypothesis that $(p_1 - p_2) = 0$ versus the hypothesis that $(p_1 - p_2) > 0$ at $\alpha = 0.05$.

$$H_0 : p_1 - p_2 = 0$$

$$H_a : p_1 - p_2 > 0$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}} = \frac{0.4 - 0.3}{\sqrt{\frac{0.4*0.6}{100} + \frac{0.3*0.7}{200}}} = 1.704$$

$$z_{.05} = 1.645$$

$1.704 > 1.645 \rightarrow$ Reject H_0

d) What assumptions did you have to make for part b) and c) above?

Random samples. No need to assume normal populations.