1. *Calibration.* Consider a simple linear regression model

$$y_i = \beta_0 + \beta_1 x_i + e_i,$$

for $i = 1, 2, ..., N$, where $e_i \sim$ iid $\mathcal{N}(0, \sigma^2)$. For notation, set $\mathbf{x} = (x_1, x_2, ..., x_N)'$ and $\mathbf{y} = (y_1, y_2, ..., y_N)'$. Suppose that a new value of y is observed, say, y_0 (independent of $y_1, y_2, ..., y_N$) and that we wish to estimate the corresponding value of x_0. Regard x_0 as a parameter.
(a) Based on the observed data $\mathbf{x}$, $\mathbf{y}$, and y_0, find the maximum likelihood estimators of β_0, β_1, x_0, and σ^2.
(b) Derive a $100(1-\alpha)$ percent confidence interval for x_0. When does such an interval exist? *Hint*: Use an $F(1, N-2)$ distribution based on $(y_0 - \widehat{y}_0)^2$, where $\widehat{y}_0 = \widehat{\beta}_0 + \widehat{\beta}_1 x_0$.

2. Consider the analysis of covariance model

$$y_{ij} = \mu_i + \gamma_i(x_{ij} - \overline{x}_{i+}) + e_{ij},$$

where $i = 1, 2, 3$ and $j = 1, 2, 3, 4$, where $e_{ij} \sim$ iid $\mathcal{N}(0, \sigma^2)$. All parameters are best regarded as fixed (unknown) constants. Derive a test statistic to test

$$H_0 : \gamma_1 = \gamma_2 = \gamma_3 = \gamma, \text{ say},$$

i.e., to test whether or not the mean response of y_{ij} is linear in x with a common slope across the 3 treatments. Simplify your test statistic as much as possible. Specify the distribution of your statistic when H_0 is true and indicate a level α rejection region.

3. Do the Exercise which follows Example 6.1 (notes, pp 132-133).

4. Use Definition 5.9 in Monahan (pp 109) to derive the probability density function of a noncentral t distribution. Also, derive the mean and variance of this distribution.

5. With the beetle data in HW6, perform each test in Example 6.3 (notes, pp 134).