DIRECTIONS:

- This exam contains two parts:
  - Part 1. 20 Multiple Choice `[40 points]`
  - Part 2. 5 Short Answer/Computation `[60 points]`
- On Part 1, circle the correct response for each question. Make sure that your answer is clearly marked. You will not receive partial credit for any work done in Part 1. On Part 2, show all of your work to receive full (or partial) credit.
- This is a closed-book examination. However, you may use three 8.5×11 sheets of notes if you wish. You may also use a calculator.
- The standard normal and t distributions are attached at the end of this examination.
- Any discussion or otherwise inappropriate communication between examinees, as well as the appearance of any unnecessary material, will be dealt with severely.
- This exam is worth a total of 100 points. **Print** your name **at the top of this page in the upper right hand corner**. *Good Luck!!*

PART 1: MULTIPLE CHOICE. Circle the correct response for each question. Make sure that your answer is clearly marked.

1. A straight-line regression model has been fit to a two-variable data set that exhibits a strong negative linear trend. The regression model is given by $\widehat{y} = 105 - 10x$. One of the data pairs is $(x, y) = (6, 50)$. What is the **residual** associated with this data pair?
(a) 45
(b) -45
(c) 5
(d) -5

2. Both the matched pairs t-test and two-sample pooled t-test are used when comparing two populations. What statement is most accurate?
(a) In a two-sample pooled t-test, the normal data assumption is not critical, whereas in the matched pairs t-test, the normal data assumption is critical.
(b) The matched pairs t-test is used only with experimental data and the two-sample pooled t-test is used only for observational study data.
(c) The two-sample pooled t test assumes that the two populations are independent. In the matched pairs t-test, the two populations are not independent.
(d) To compute the two sample t-test statistic, we use the data differences from both populations. The matched pairs t statistic assumes a known population standard deviation.

3. The random variable X has a normal distribution with mean $\mu = 15$ and standard deviation $\sigma = 4$. Which of the following is closest to $P(X > 18)$?
(a) 0.23
(b) 0.67
(c) 0.75
(d) 0.77

4. In statistics, a procedure is said to be **robust** if
(a) the results obtained from the procedure are not heavily dependent on the procedure's assumptions.
(b) the statistic used in the procedure is normally distributed.
(c) probability values computed using the procedure are less than α.
(d) the procedure has the highest power when the null hypothesis is true.

5. True or False. The binomial distribution is a discrete probability distribution.
(a) True
(b) False

6. Consider the following straight-line regression model, obtained from an SRS of first-year students at a small school in Menomonie, WI.

$$\widehat{\text{GPA}} = -2.22 + 0.045(\text{IQ})$$

The r^2 statistic from the data was computed to be 0.812 (or 81.2 as a percent). What is the interpretation of this r^2 statistic?
(a) Since the r^2 is pretty high for this regression, the model must be a good model for the data.
(b) We can be approximately 81 percent confident that our predictions will include the mean GPA.
(c) About 81 percent of the data values fall on the regression line.
(d) Roughly 81 percent of the variation in the GPA variable is explained by the IQ variable.

7. An experiment is performed and a variable is measured. The **sample space** for the experiment refers to
(a) the density curve associated with the variable measured.
(b) the set of all possible outcomes that can be observed for the variable.
(c) the sampling distribution of a statistic computed for the data.
(d) the quality of the randomization used in the experiment.

8. True or False. The **Law of Large Numbers** says (loosely speaking) that the sampling distribution of the sample mean is normal (and the approximation improves with larger sample sizes).
(a) True
(b) False

9. Which most accurately describes the **stratified random sampling** model?
(a) It is a scheme where each sample of size n has an equal chance of being selected.
(b) The population is first divided up into groups of similar characteristics. Then simple random samples are taken from each group.
(c) The population is divided up into subpopulations, a random sample of subpopulations is chosen, and each individual from the chosen subpopulations is surveyed.
(d) The frame for the study is divided up into n lists. One individual is chosen from the specific position in the first list. Then, from the same position, one individual is chosen from the remaining $n-1$ lists.

10. True or False. Two events are said to be **disjoint** if their probabilities, when multiplied together, give the probability of seeing both events occur simultaneously.
(a) True
(b) False

11. What is **Type I Error**?
(a) It is the probability of observing a test statistic as extreme as the one you observed.
(b) It is the event that the researcher rejects a true null hypothesis.
(c) It is the probability of not rejecting H_1.
(d) It occurs when the researcher does not reject a null hypothesis that is false.

12. What is a **probability value**?
(a) It is the probability of observing a test statistic as extreme as the one you observed.
(b) It is the probability of rejecting H_0.
(c) It is the probability of not rejecting H_1.
(d) The value for which a hypothesis test has maximum power.

13. A statistic is said to be an **unbiased estimator** for a population parameter if
(a) its sampling distribution is approximately normal.
(b) the statistic can be computed in repeated sampling.
(c) its sampling distribution has mean equal to the parameter.
(d) the standard deviation of the statistic is equal to the parameter.

14. When describing relationships with the correlation, often times people are careless. Which one of the following statements does **not** contain a mistake?
(a) "The correlation between gender and promotion rates was significantly higher than zero. This suggests that males and females are promoted at different rates."
(b) "The correlation between age and number of failures was much stronger for the experimental formulation ($r = 0.35$) than it was for the control formulation ($r = -0.86$)."
(c) "The correlation between yield (in kg) and amount of fertilizer (sodium nitrate, in kg) was close to zero. Hence, there is virtually no linear relationship between the yield and the amount of sodium nitrate applied."
(d) "For our experiment, our analysis showed the correlation was $r = 0.78$ pounds."

15. In computing a t-confidence interval for μ, you note that the total length of the interval is equal to 10. What is the **margin of error**?
(a) 5
(b) 10
(c) 20
(d) This can not be determined since σ is not known.

16. After fitting a least-squares regression line to data, you display the residual plot. The residual plot appears to have two patterns: a fan-shape pattern and a curved trend. What does this suggest?
(a) The scatterplot of the data probably shows a negative correlation.
(b) The scatterplot of the original data probably has a random appearance.
(c) The straight line model is inadequate for the data.
(d) The straight line model is adequate for the data.

17. I want to obtain some information on the party affiliation of individuals in Connecticut. I contact 1000 individuals, and 439 of the individuals were identified as "Democrat." In this setting, what is the **sample**?
(a) All residents of Connecticut.
(b) All residents of Connecticut that are registered to vote.
(c) The 1000 individuals contacted.
(d) The 439 individuals that were identified as "Democrat."

18. The following statements appeared on a television commercial that I saw recently:

> "For the 120 individuals, the average amount of money saved was 5,418 dollars. The range was from 1,203 dollars to 251,989 dollars."

Which one of the following statements can **not** be true?
(a) The standard deviation for the 120 individuals is equal to zero.
(b) The median for the 120 individuals is smaller than 5,418 dollars.
(c) The third quartile for the 120 individuals is smaller than 5,418 dollars.
(d) The first quartile for the 120 individuals is smaller than 5,418 dollars.

19. Suppose that X has a **binomial** distribution with $n = 10$ and $p = 0.2$. What is $P(X = 1)$?
(a) 0.107
(b) 0.268
(c) 0.732
(d) 0.893

20. What is the 90th percentile of the **standard normal distribution**?
(a) 0.84
(b) 1.28
(c) 1.65
(d) 1.96

PART 2: SHORT ANSWER/COMPUTATION. Show all of your work, and explain your reasoning.

1. A certain species of beetle produces offspring with either blue eyes or black eyes. A biologist wants to determine which, if either, of the two eye colors is dominant for this species of beetles. Let p denote the proportion of offspring that possesses blue eyes. A simple random sample of $n = 100$ beetles reveals that 44 of the beetles possess blue eyes.

(a) If, in fact, the beetles produce blue-eyed and black-eyed offspring at an equal rate, then $p = 0.5$. Thus, the biologist wants to test $H_0 : p = 0.5$ versus $H_1 : p \neq 0.5$. Conduct this test using $\alpha = 0.10$. Show all of your calculations and state all conclusions **in terms of the problem**!

(b) In a larger study, suppose that the biologist wants to get a 95 percent confidence interval for p with a margin of error of $m = 0.02$. How many beetles would the biologist need to observe to attain these goals?

2. A client came to me with data from a meteorological study that compared two temperature measurement methods: a manual approach (MAN) and an automated approach (AUTO). In 1990, temperature measurements were made using the manual approach at a location in Greensboro, NC. In 1997, temperature measurements were made using the automated approach at a location in Raleigh, NC. The objective of the study was to determine if there was a difference between the two measurement methods (MAN versus AUTO).

The client provided me with two data sets. The first data set was the 365 daily high temperatures in Greensboro in 1990 (measurements were made using the MAN method), and the second data set was the 365 daily high temperatures in Raleigh in 1997 (measurements were made using the AUTO method).

I told the client that he could not compare the measurement methods (MAN versus AUTO) with the data that he gave me.

(a) Suppose that I had compared the data using a two-sample t-test. There are two lurking variables that I can not account for in my comparison. What are they? Because of the lurking variables, a significant test statistic would have told me nothing about the two measurement methods.

(b) You have been put in charge of developing a **randomized complete block design (RCBD)** that would help the meterologist compare the measurement methods. The two locations, Greensboro and Raleigh, will serve as blocks. Clearly illustrate how you would design the experiment. For example, address issues like

- What would be the response variable?
- When and where should you conduct the study?
- Who would conduct the experiment?
- How long will the study last?

(c) We haven't talked about how to analyze data from a RCBD, but the interpretation of probability values is the same. Clearly explain what the following sentence means.

> "The analysis from the RCBD revealed that the measurement method variable (MAN versus AUTO) was not significant ($P > 0.20$)."

Use the next page for your answers.

This is a blank page for Problem 2.

3. In an SRS, we observe data $x_1, x_2, ..., x_n$. Assume that these data are **normally distributed** with mean μ and standard deviation σ. Here, μ and σ are **unknown**.

(a) What is the sampling distribution of the sample mean $\overline{x}$?

(b) Give the sampling distribution of

$$\frac{\overline{x}-\mu}{\sigma/\sqrt{n}}. \qquad \text{ANSWER:}$$

(c) Let s denote the **sample standard deviation**. Give the sampling distribution of

$$\frac{\overline{x}-\mu}{s/\sqrt{n}}. \qquad \text{ANSWER:}$$

(d) Suppose that goal is to estimate the **population standard deviation** σ. With our n data values, we can compute the sample standard deviation, which, recall is

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}.$$

A QUICK ESTIMATE: Consider **another approach** to estimating the population standard deviation σ. We know that since the population distribution is normal, almost all of the data values $x_1, x_2, ..., x_n$ should fall within 3 standard deviations from the mean (Empirical Rule). Furthermore, we know that all of the data fall between x_{min} and x_{max} (the smallest and largest values, respectively). Recall that the **sample range** is defined by $R = x_{max} - x_{min}$. Thus, it should seem reasonable that $R \approx 6\sigma$. Solving for σ, one obtains

$$\sigma \approx R/6.$$

This last formula provides a "quick way" of estimating the standard deviation σ, since all one has to compute is the sample range R. It is computationally much easier than using the sample standard deviation s.

(e) Suppose that in a manufacturing setting, the distribution of the length of a certain part is **normal** with mean μ and standard deviation σ. Here are the lengths of $n = 6$ randomly selected parts (measured in mm).

```
28.7    34.4    33.4    30.4    32.8    32.3
```

Compute two estimates of σ, one using the sample standard deviation, and one using the $\sigma \approx R/6$ approximation. Show all of your work. You may use a calculator to check your answers, but I want to see all of your calculations.

(f) Treating these 6 parts as an SRS from a large population, find a 99 percent confidence interval for μ, the population mean length.

This is a blank page for Problem 3.

4. Many species of terrestrial frogs that hibernate at or near the ground surface can survive prolonged exposure to low winter temperatures. In freezing conditions, the frog's body temperature, some referred to as its *supercooling temperature*, remains relatively higher than outside temperatures due to an accumulation of glycerol in its body fluids. Studies have shown that the supercooling temperature (under mildly freezing conditions) of many terrestrial frogs is **normally distributed**.

Suppose that a researcher's goal is to estimate μ, the mean supercooling temperature for the Northern Leopard Frog (found in the northern New England states). The researcher's has selected $n = 68$ frogs (using random sampling) and has subjected each frog to a prolonged exposure, in a controlled environment, at temperature -1 degrees Celcius (typical mildly freezing conditions).

Our researcher would like to test whether or not the mean supercooling temperature for the Northern Leopard species is significantly different than 2.1 degrees. He uses a **two-sided test** because he does not know if μ will be larger than 2.1 or smaller than 2.1. Here is the output from the study:

```
Test of mu = 2.1 vs not = 2.1

Variable     N    Mean   StDev  SE Mean        95% CI            T       P
super_temp  68  1.1886  3.8621   0.4683  (0.2537, 2.1234)  -1.95   0.056
```

Our researcher knows how to use Minitab, but that is about it. Explain the output to the researcher (explain each piece of it). And, help the researcher understand what the output is saying about the Northern Leopard frog population. Use the back of this page if necessary.

Note: Our researcher does not understand things like "reject H_0" and "do not reject H_0!!" (he didn't have my statistics class). Keep this in mind!

5. You are working at General Electric and are participating in a study that is examining the starting time of jet engines (y, measured in seconds) and the amount of thrust exerted (x, measured in a scale similar to "horsepower"). Thrust is the measurement of engine power. The higher the thrust, the more the power. There were two types of engines used in the study: Type A engines and Type B engines. Here are the data from the study.

Type	Thrust (x)	Time (y)	Type	Thrust (x)	Time (y)
A	584	2.46	B	697	2.10
A	606	2.44	B	671	2.15
A	732	2.15	B	657	2.20
A	645	2.38	B	740	2.05
A	700	2.19	B	713	2.06
A	598	2.44	B	705	2.09
A	664	2.26	B	680	2.13
A	696	2.22	B	741	2.02
A	634	2.34	B	687	2.10
A	679	2.30	B	749	2.01

A scatterplot of the data appears below in Figure 1. I have also fit two straight-line regressions; one for the Type A engines (solid line) and one for the Type B engines (dotted line). I have used plotting symbols "A" and "B" to distinguish the two types of engines. Four questions appear on the next page.

Four questions appear on the next page.

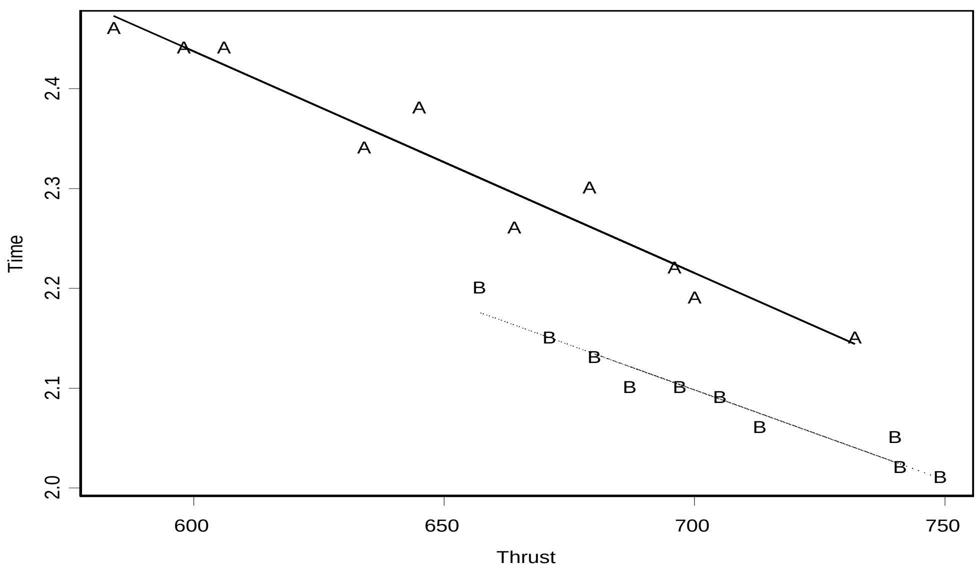

Figure 1: *Thrust-time data for two different engine types.*

The least-squares regression line **for the Type A engines** is given by

$$\widehat{\text{TIME}} = 3.7698 - 0.0022 \times (\text{THRUST}).$$

(a) What is our estimate of the **slope** for the Type A engines?

(b) What would you predict for the starting time of Type A engines which exerted a thrust reading of 625? Show your work.

(c) Your boss wants you to make the same prediction of starting time for engines which exerted a thrust reading of 625, except **now** for Type B engines. What would you report to your boss?

(d) Take the 10 Type A engines to be an SRS of all Type A engines, and similarly for the 10 Type B engines. Suppose that you are interested in testing

$$H_0 : \mu_A - \mu_B = 0$$
$$\text{versus}$$
$$H_1 : \mu_A - \mu_B < 0,$$

where μ_A is the mean thrust for Type A engines, and μ_B is the mean thrust for Type B engines. The value of the **pooled** two-sample t statistic needed for this test is $t = -2.73$. **You don't have to verify this calculation!!**

Is this a statistically significant result at $\alpha = 0.05$? What about at $\alpha = 0.01$? Explain your answers very clearly. Use the back of this page if necessary.