1. Do the following problems in Chapter 2 from Cryer and Chan: 2.5, 2.7, 2.10, 2.11, 2.12, 2.19, and 2.27. Although not always stated, it is understood that $\{e_t\}$ is a zero mean white noise process with $\text{var}(e_t) = \sigma_e^2$.

2. Suppose that Z_1 and Z_2 are uncorrelated random variables with zero mean and unit variance. Consider the process defined by

$$Y_t = Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t$$

where $e_t \sim \text{iid } \mathcal{N}(0, \sigma_e^2)$ and $\{e_t\}$ is independent of both Z_1 and Z_2.
(a) Prove that $\{Y_t\}$ is stationary.
(b) Let Z_1 and Z_2 be independent $\mathcal{N}(0,1)$ random variables, and set $\sigma_e^2 = 1$ and $\omega = 0.5$. Use the following R commands to simulate $n = 150$ observations from the $\{Y_t\}$ process:

```
> omega = 1
> Z = rnorm(2,0,1)
> e.t = rnorm(150,0,1)
> Y.t = e.t*0
> for (i in 1:length(e.t)){
      Y.t[i] = Z[1]*cos(omega*i) + Z[2]*sin(omega*i) + e.t[i]}
> plot(Y.t,ylab="Trigonometric process",xlab="Time",type="o")
```

Describe the appearance of your time series.
(c) Amend the R code above to simulate a realization of the process

$$\widetilde{Y}_t = \beta_0 + \beta_1 t + Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t,$$

where $\beta_0 = 100$, $\beta_1 = 0.05$, $\sigma_e^2 = 1$ and $\omega = 0.5$. To do this, replace the last three lines of the R code above with

```
> Y.tilde = e.t*0
> for (i in 1:length(e.t)){
   Y.tilde[i] = 100 + 0.05*i + Z[1]*cos(omega*i) + Z[2]*sin(omega*i) + e.t[i]}
> plot(Y.tilde,ylab="Trig process with linear trend",xlab="Time",type="o")
```

Does your $\{\widetilde{Y}_t\}$ process appear to be stationary? What is the effect of adding the linear trend term $\beta_0 + \beta_1 t$ to the model?
(d) Plot the first differences of your simulated $\{\widetilde{Y}_t\}$ process. To do this, use

```
> diff.Y.tilde = diff(Y.tilde)
> plot(diff.Y.tilde,ylab="First differences of Y.tilde",xlab="Time",type="o")
```

Describe the appearance of this first difference process $\{\nabla\widetilde{Y}_t\}$. In particular, does it appear to be stationary in the mean level? Are you surprised? Discuss the behavior of

each process: $\{Y_t\}$, $\{\widetilde{Y}_t\}$, and $\{\nabla\widetilde{Y}_t\}$, and how they relate to each other.
(e) In part (c), suppose that instead of the added linear trend $\beta_0 + \beta_1 t$, the added trend was quadratic, say, $\beta_0 + \beta_1 t + \beta_2 t^2$. Do you think the first differences of the quadratic trend version of the process in part (c) would be stationary? Explain your intuition.

3. Give an example of a process $\{Y_t\}$ that satisfies the following. *In each part, prove that your answer is correct.*

(i) A process with constant mean but variance that increases with time.

(ii) A stationary process whose autocovariance does not go to zero as time lag goes to infinity.

(iii) A nonstationary process whose autocovariance depends only on time lag.

(iv) A stationary process that has nonzero autocorrelation only at lag $k = 1$.

(v) A stationary process that has nonzero autocorrelation only at lag $k = 4$.

(vi) A nonstationary process whose first differences are stationary.