**Ground rules:** You must work alone on this quiz. Do not collaborate with anyone, either within or outside the class, to obtain answers or even hints. Questions of clarification should be directed to the instructor; in addition, the use of the Internet should be avoided. This quiz contains 6 questions, each of equal weight, making the quiz worth 60 points.

1. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid $\mathcal{N}(\mu, \sigma^2)$ sample, where $\sigma^2 = 9$, and that we would like to test $H_0 : \mu = 10$ versus $H_a : \mu > 10$. We will use a rejection region of the form $\text{RR} = \{\boldsymbol{y} : \overline{y} > k\}$.
(a) Find the values of $n$ and $k$ that provide a Type I Error probability of $\alpha = 0.02$ and a Type II Error probability, when $\mu = 12$, of $\beta = 0.1$.
(b) With your values of $n$ and $k$ from part (a), graph the power function $K(\mu)$. Try to use R to plot the function.

2. Suppose that $Y \sim \mathcal{U}(-\theta, \theta)$, where $\theta > 0$, and that we would like to test $H_0 : \theta = 1$ versus $H_a : \theta > 1$, based on the value of $Y$ using the rejection region $\text{RR} = \{y : |y| > k\}$.
(a) Find the value of $k$ that provides a level $\alpha = 0.05$ test.
(b) Using your rejection region from part (a), graph the power function $K(\theta)$. What is $K(1.5)$?

3. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid sample of size $n$ from $f_Y(y; \theta)$, where

$$f_Y(y;\theta) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

To test $H_0 : \theta = 1$ versus $H_a : \theta > 1$, suppose that we use the rejection region

$$\text{RR} = \left\{ \boldsymbol{y} : \prod_{i=1}^{n} y_i > k \right\},$$

that is, we will reject $H_0$ in favour of $H_a$ whenever the product $T = \prod_{i=1}^{n} Y_i$ exceeds $k$.
(a) Show that $T$ is a sufficient statistic for $\theta$.
(b) Find an exact expression for $k$ that gives a Type I Error probability equal to $\alpha$. *Hint:* Consider $-\log T$.
(c) Rewrite the hypothesis test above in terms of the population mean $\mu$; i.e., if $\theta = 1$, what is $\mu$? Then redo (b) using an asymptotic-based approximation for a single population mean (the rejection region will be different). Compare numerically the two critical values for different values of $n$ and $\alpha = 0.05$. As $n$ gets larger, what happens?

4. Is there an exceptionally high percentage of the executives of large corporations that are right-handed? It is assumed that 85 percent of the general population is right-handed. In a random sample of 60 chief executive officers, it was found that 57 were right-handed. Is this a statistically significant result at the $\alpha = 0.05$ level? Answer this question by performing an exact test (based on the binomial distribution) and a large-sample test

based on the normal approximation. Do the two tests provide different conclusions?

5. An educator conducted an experiment to test whether new directed reading activities in the classroom will help elementary school pupils improve some aspects of their reading ability. She arranged for a third grade class of 21 students to follow these activities for an 8-week period. A control classroom of 23 third graders followed the same curriculum without the activities. At the end of the 8 weeks, all students took a Degree of Reading Power (DRP) test, which measures the aspects of reading ability that the treatment is designed to improve. Here are the test scores.

```
Treated:  24 43 58 71 43 49 61 44 67 49 53 56 59 52 62 54 57 33 46 43 57
Control:  42 43 55 26 62 37 33 41 19 54 20 85 46 10 17 60 53 42 37 42 55
          28 48
```

The educator wants to compare the groups of pupils. Analyse these data, stating any assumptions you make along the way.

6. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid $\mathcal{N}(0, \sigma^2)$ sample, where $\sigma^2 > 0$ is unknown. We are interested in testing $H_0 : \sigma^2 = \sigma_0^2$ versus $H_1 : \sigma^2 < \sigma_0^2$, where $\sigma_0^2$ is a fixed constant. We will use a rejection region which has the following form:

$$\text{RR} = \left\{ \boldsymbol{y} : \sum_{i=1}^{n} y_i^2 < k \right\}.$$

(a) Show that this rejection region depends on the sufficient statistic for $\sigma^2$.
(b) Determine the value of $k$ so that the rejection region is of level $\alpha$.
(c) For $n = 20$, $\alpha = 0.10$, and $\sigma_0^2 = 4$, and using your rejection region in part (b), graph the power function $K(\sigma^2)$. Try to use R to plot the function.