

Here is a summary of some of the main ideas from the prerequisite STAT/MATH 511: Probability. WMS stands for Wackerly, Mendenhall, and Schaeffer (our text).

Chapter 1 (WMS): Overview of basic statistics. We skipped this chapter (STAT 110).

Chapter 2 (WMS): Basic probability theory, set notation, Kolmogorov axioms, complement rule, tools for counting (e.g., combinations, permutations, etc.), conditional probability, Law of Total Probability, Bayes Rule.

Chapter 3 (WMS): Discrete random variables (positive probability is assigned to specific points), probability mass functions (pmf), means and variances of discrete random variables, moment generating functions, Bernoulli trials. Important discrete models:

1. Discrete uniform. Equal probability assigned to each support point.
2. Binomial, $b(n, p)$ (Bernoulli, $n = 1$). Number of successes out of n Bernoulli trials.
3. Geometric, $\text{geom}(p)$. Number of Bernoulli trials until 1st success.
4. Negative binomial, $\text{nib}(r, p)$. Number of Bernoulli trials until the r th success; generalization of the geometric.
5. Hypergeometric, $\text{hyper}(N, n, r)$. Number of Class 1 objects selected from r . Finite population version of the binomial.
6. Poisson, $\text{Poisson}(\lambda)$. Records counts in a Poisson process over time or space.

Chapter 4 (WMS): Continuous random variables (positive probability is assigned to intervals; not specific points), probability density functions (pdf), cumulative distribution functions (cdf), means and variances of continuous random variables, moment generating functions, Chebyshev. Important continuous models:

1. Uniform, $\mathcal{U}(\theta_1, \theta_2)$. Pdf is constant over the interval from θ_1 to θ_2 .
2. Normal, $\mathcal{N}(\mu, \sigma^2)$. Most widely used probability model. $E(Y) = \mu$ and $V(Y) = \sigma^2$. Symmetric, unimodal, “bell-shaped.”

3. Gamma, $\text{gamma}(\alpha, \beta)$. Popular model for random variables with positive support. Shape parameter, α ; scale parameter, β . Skewed right, in general.
4. Exponential, $\text{exponential}(\beta)$. A gamma distribution with $\alpha = 1$. Exponential-decay shaped pdf.
5. χ^2 , $\chi^2(\nu)$. A gamma distribution with $\alpha = \nu/2$ and $\beta = 2$. Degrees of freedom parameter, ν . Popular model in applied statistics.
6. Beta, $\text{beta}(\alpha, \beta)$. Support over $(0, 1)$. Very flexible model for proportions.
7. Other “named” distributions: Cauchy, Weibull, log-normal, Pareto, etc.

Chapter 5 (WMS): Random vectors, multivariate distributions (particular attention paid to bivariate distributions), joint pmfs and pdfs, marginal distributions, conditional distributions, independence, multivariate expectations, covariance/correlation, multinomial distribution, bivariate normal, conditional expectations, iterated rules for mean and variance.

Going forward: It will be helpful to know all of the named discrete/continuous distributions (that we discussed in STAT/MATH 511), their pmfs/pdfs, means, variances, and moment generating functions. Recall that the cumulative distribution function for a random variable Y is given by

$$F_Y(y) = P(Y \leq y).$$

Useful integral shortcuts: For $\alpha > 0$ and $\beta > 0$, recall that

$$\int_0^{\infty} y^{\alpha-1} e^{-y/\beta} dy = \Gamma(\alpha)\beta^{\alpha}$$

and

$$\int_0^1 y^{\alpha-1} (1-y)^{\beta-1} dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$

These facts follow from the properties of gamma and beta pdfs, respectively. Recall that the gamma function $\Gamma(s)$ satisfies $\Gamma(s) = (s-1)\Gamma(s-1)$, for $s > 1$.