GROUND RULES:

- This exam contains 5 questions; each question is worth 10 points. The maximum number of points on this exam is 50.
- Print your name at the top of this page in the upper right hand corner.
- This is a closed-book and closed-notes exam. You may use a calculator if you wish.
- SHOW ALL OF YOUR WORK AND EXPLAIN ALL OF YOUR REASONING!!! **Correct answers with no explanation receive almost no credit.**
- Summary information on the discrete and continuous distributions is provided.
- Any discussion or otherwise inappropriate communication between examinees, as well as the appearance of any unnecessary material, will be dealt with severely.
- You have 80 minutes to complete this exam. GOOD LUCK!

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

> *I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.*

1. Suppose that $Y \sim \text{gamma}(\alpha, \beta)$. Define

$$U = g(Y) = Y^2 - 1.$$

(a) Find the probability density function (pdf) of U. Make sure to note the support.
(b) Find $E(U)$.

2. For a certain insurance policy, actuaries model the claim amount Y (measured in thousands of dollars) using a Pareto distribution with pdf

$$f_Y(y) = \begin{cases} \dfrac{24}{y^4}, & y > 2 \\ 0, & \text{otherwise.} \end{cases}$$

An iid sample $Y_1, Y_2, ..., Y_5$ of $n = 5$ claims is observed.
(a) Find the probability that the maximum claim exceeds \$4,000; i.e., find $P(Y_{(5)} > 4)$.
(b) Find the expected value of the minimum claim amount.

3. Suppose that Y_1 and Y_2 are random variables with joint pdf

$$f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} 4y_1y_2, & 0 < y_1 < 1,\ 0 < y_2 < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the pdf of $U = Y_1^2 Y_2$. *Advice:* Do not try to use mgfs.
(b) Find $E(U)$.

4. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid sample of exponential($\beta = 1$) random variables. Define the statistics

$$\begin{aligned} U_n &= 2(Y_1 + Y_2 + \cdots + Y_n) \\ V_n &= \sqrt{n}(\overline{Y} - 1). \end{aligned}$$

(a) Derive the moment generating function (mgf) of U_n. What is the distribution of U_n?
(b) Show that the mgf of V_n, for $t < \sqrt{n}$, is

$$m_{V_n}(t) = \{e^{t/\sqrt{n}} - (t/\sqrt{n})e^{t/\sqrt{n}}\}^{-n}.$$

(c) For any $t < \sqrt{n}$, find $\lim_{n\to\infty} m_{V_n}(t)$. *Hint:* Think about the CLT when you are examining the V_n sequence.

5. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid $\mathcal{N}(0, \sigma^2)$ sample. Let $\overline{Y}$ and S^2 denote the usual sample mean and sample variance, that is,

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \quad \text{and} \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2.$$

(a) State the sampling distribution of $\overline{Y}$.
(b) Find $V(S^2)$.
(c) Define

$$Q_1 = \frac{1}{\sigma^2}\left[n\overline{Y}^2 + (n-1)S^2\right] \quad \text{and} \quad Q_2 = \frac{n\overline{Y}^2}{S^2}.$$

Argue that $Q_1 \sim \chi^2(n)$ and that $Q_2 \sim F(1, n-1)$. These arguments need not be overly mathematical, but they must be convincing.