GROUND RULES:

- This exam contains 4 questions; each question is worth 10 points. The maximum number of points on this exam is 40.
- Print your name **at the top of this page in the upper right hand corner**.
- This is a closed-book and closed-notes exam. You may use a calculator if you wish, but SHOW ALL OF YOUR WORK AND EXPLAIN ALL OF YOUR REASONING!!!
- Summary information on the discrete and continuous distributions is provided.
- A standard normal table is provided.
- Any discussion or otherwise inappropriate communication between examinees, as well as the appearance of any unnecessary material, will be dealt with severely.
- You have 55 minutes to complete this exam. GOOD LUCK!

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

> *I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.*

1. Suppose that $Y_1, Y_2, ..., Y_5$ is an iid sample of size $n = 5$ from a $\mathcal{N}(0, 1)$ distribution. Let $\overline{Y}$ and S^2 denote the sample mean and sample variance, respectively. Define the statistics

$$\begin{aligned} T_1 &= \frac{Y_1^2}{(Y_2^2 + Y_3^2)/2} \\ T_2 &= \frac{\overline{Y}}{\sqrt{S^2/5}} \\ T_3 &= 4S^2 + (\sqrt{5}\overline{Y})^2 \end{aligned}$$

Give the precise distribution of each statistic. **You need not derive anything here rigourously; you can use sampling distribution results we discussed/proved in class. However, your arguments should be convincing. A correct answer without explanation receives almost no credit.**

2. Let Y denote the time (in minutes) that it takes for a customer representative to respond to a telephone inquiry. It is assumed that Y follows a **uniform** distribution from 0.5 to 3.5. That is, $Y \sim \mathcal{U}(0.5, 3.5)$.

In a 2-hour shift, the customer representative responds to $n = 15$ calls with response times $Y_1, Y_2, ..., Y_{15}$. Treating these times as an iid sample, approximate the probability that the sample mean response time $\overline{Y}$ is less than 2.15 minutes. That is, approximate $P(\overline{Y} < 2.15)$. *Hint:* Use CLT.

3. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid sample from a $\mathcal{N}(\mu, \sigma^2)$ population, where both μ and σ^2 are unknown parameters. To estimate σ^2, we will use an estimator of the form $\widehat{\sigma}^2 = cS^2$, where S^2 is the usual sample variance; that is,

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2,$$

and $c > 0$ is a strictly positive constant. Prove that the value of c that minimises $\text{MSE}(\widehat{\sigma}^2) = \text{MSE}(cS^2)$ is given by

$$c = \frac{n-1}{n+1}.$$

Hint: Recall that $\text{MSE}(cS^2) = V(cS^2) + [B(cS^2)]^2$. Even if you can't get the right answer, explain clearly your approach.

4. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid Bernoulli(p) sample, where $0 < p < 1$, and let $\widehat{p}$ denote the usual sample proportion. In class, we have shown that

$$\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}},$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ quantile from the standard normal distribution, is an approximate $100(1-\alpha)$ percent confidence interval for p.

The inconsistencies associated with this interval are well documented, so, in this problem, we will find an alternative interval for p. The construction of the alternative interval is based on the fact that

$$h(\widehat{p}) \equiv \arcsin(\widehat{p}^{1/2}) \;\sim \mathcal{AN}\left[\arcsin(p^{1/2}), \frac{1}{4n}\right],$$

when n is large.

(a) Use the last fact to construct a large-sample $100(1-\alpha)$ percent confidence interval for

$$h(p) \equiv \arcsin(p^{1/2}).$$

Define any notation you use. *Hint:* Consider

$$Q = \frac{h(\widehat{p}) - h(p)}{\sqrt{1/4n}}.$$

(b) Transform the endpoints of the interval in (a) to produce a large-sample $100(1-\alpha)$ percent confidence interval for the parameter p. You do this by finding the inverse function $h^{-1}(p)$ and applying this inverse rule to the endpoints of the $h(p)$ interval. Recall that $\arcsin(\cdot)$ is another symbol for $\sin^{-1}(\cdot)$.

This is an extra page for Problem 4. Use it if you wish.