Ground rules: You must work alone on this quiz. Do not collaborate with anyone, either within or outside the class, to obtain answers or even hints. Questions of clarification should be directed to the instructor; in addition, the use of the Internet should be avoided. This quiz contains 7 questions, each of equal weight, making the quiz worth 70 points.

1. On the first day of class, we discussed the underlying notion of a random experiment, the associated sample space, and assigning probabilities to events in the sample space. Describe a phenomena that will happen to you soon (e.g., this weekend, this year, etc.) and conceptualise it as a random experiment. Describe your associated sample space, events of interest to which you might like to assign probability (using appropriate set notation), and a procedure for assigning probabilities to events. **I'm looking for creativity here!** Don't give me a vacuous experiment like flipping a coin.

2. Suppose that we have n identical white balls, numbered $1, 2, ..., n$, in one drum. Suppose that we have m identical red balls, numbered $1, 2, ..., m$, in a second drum. Consider the following experiment of

- choosing r balls from the first drum and observing the ball numbers, AND
- choosing s balls from the second drum and observing the ball numbers.

That is, balls are drawn from each drum. In addition, balls are drawn from each drum at random and **without replacement**; i.e., balls are not replaced after they are selected.
(a) Describe a sample space for this experiment, which, within colour, does not regard the ordering of the balls drawn as important. Each sample point should be a vector of length $r + s$, corresponding to the $r + s$ numbers chosen.
(b) Suppose that we pick a sample point at random from the underlying sample space. What is the probability associated with this point? What assumptions are you making?
(c) Evaluate your expression in part (b) when $r = 5$, $n = 55$, $s = 1$, and $m = 42$. If your answer in part (b) is correct, your answer here is the probability of winning the Powerball lottery!

3. Suppose that an experiment is to be performed where the sample space $S = (0, 1)$ and the probability measure P assigns probabilities to events $A \subset S$ using the rule

$$P(A) = \int_A f(y)dy,$$

where $f(y) = 6y(1 - y)$, for $0 < y < 1$, and $f(y) = 0$, otherwise. Prove that the probability measure P satisfies the three Kolmolgorov axioms.

4. There were seven accidents in a town during a seven-day period. Define A to be the event that all seven accidents occurred on the same day. Define B to be the event that

each of the seven accidents occurred on a different day. Find $P(A)$ and $P(B)$. What assumptions are you making?

5. An insurance company examines its pool of auto insurance customers and gathers the following information:

- All customers insure at least one car.
- 70 percent of the customers insure more than one car.
- 20 percent of the customers insure a sports car.
- Of those customers who insure more than one car, 15 percent insure a sports car.

(a) Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.
(b) Calculate the probability that a randomly selected customer insures more than one car, given that s/he insures a sports car.

6. Suppose that A, B, and C are events in a nonempty sample space S. Prove each of the following facts:
(a) If $P(A|B) = P(A|\overline{B})$, then A and B are independent.
(b) If $P(A|C) > P(B|C)$ and $P(A|\overline{C}) > P(B|\overline{C})$, then $P(A) > P(B)$.
(c) $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$.

7. A transmitter is sending a message in binary code ("+" and "−" signals) that must pass through two independent relay stations before being sent on to the receiver. Schematically, the message is sent as follows

$$\text{Transmitter} \implies \text{Relay 1} \implies \text{Relay 2} \implies \text{Receiver.}$$

At each relay station, there is a 25 percent chance that a signal will be reversed; that is, for $i = 1, 2$,

$$\begin{aligned} P(\text{“}+\text{” is sent by relay } i | \text{“}-\text{” is received by relay } i) &= 0.25 \\ P(\text{“}-\text{” is sent by relay } i | \text{“}+\text{” is received by relay } i) &= 0.25. \end{aligned}$$

Suppose that "+" symbols make up 60 percent of the messages being sent by the transmitter. If a "+" is received from Relay 2, what is the probability that a "+" was sent by the transmitter?