

GROUND RULES:

- This exam contains 5 questions; each question is worth 10 points. The maximum number of points on this exam is 50.
- Print your name **at the top of this page in the upper right hand corner.**
- This is a closed-book and closed-notes exam. I will provide you with the discrete/continuous distribution summaries as promised.
- You may use a calculator if you wish, but **SHOW ALL OF YOUR WORK AND EXPLAIN ALL OF YOUR REASONING!!!**
- Any discussion or otherwise inappropriate communication between examinees, as well as the appearance of any unnecessary material, will be dealt with severely.
- You have 60 minutes to complete this exam. **GOOD LUCK!**

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.

1. A phase II clinical trial, currently being conducted in Bethesda, MD, is examining how well ixabepilone works in treating patients with renal cell carcinoma (kidney cancer). Patients are currently being enrolled in the trial. For this problem, we will assume that 35 percent of all patients will respond positively to ixabepilone.

(a) Let Y denote the number of patients in the trial who respond positively to ixabepilone (e.g., the drug reduces tumor size). Clearly explain, in the context of the problem, what assumptions must be true for Y to have a binomial distribution.

(b) Suppose now that patients will be enrolled continually in the trial until the 30th patient responds positively to ixabepilone. What is the probability that no more than 100 patients will be needed? Write an expression (in terms of a pmf/pdf) that gives this probability. Do not evaluate this expression numerically.

(c) At the end of the recruitment period, suppose that there are 90 patients. Ten of these patients are HIV positive. If we select a random sample of 5 patients from the 90 (without replacement), what is the probability that exactly one is HIV positive? Here, I am looking for a numerical answer.

2. Let Y denote the number of calls received per day by the USC Campus Police. Suppose that Y has a Poisson distribution with mean 6.

- (a) What is the probability that, on a given day, there are at most 2 calls?
(b) Suppose that the daily cost (in dollars) to respond to Y calls is given by

$$C = 150 + 100Y$$

Find the expected value and variance of C .

- (c) Starting at the beginning of the day, let T denote the time until the second call is received. What is the distribution of T ? Be specific.

3. Suppose Y is a continuous random variable with probability density function (pdf)

$$f_Y(y) = \begin{cases} ky, & 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of k and then graph the pdf.
(b) Find the cumulative distribution function (cdf) of Y and graph it. Remember to consider the different cases (there are three).

4. Suppose that Y has a beta distribution with parameters $\alpha > 0$ and $\beta = 2$.

(a) Show that

$$E(\sqrt{Y}) = \frac{\alpha(\alpha + 1)}{(\alpha + \frac{1}{2})(\alpha + \frac{3}{2})}.$$

(b) Find $\phi_{0.5}$, the median of Y , when $\alpha = 1$ and $\beta = 2$. Other names for the median include the $p = 0.5$ quantile or the 50th percentile.

5. Define Y to be the time (in years) from purchase until failure of a \$30,000 piece of manufacturing equipment. Engineers have determined that Y follows an exponential distribution with mean 5.

(a) Find the probability that the piece of equipment will fail before 2 years after purchase.

(b) The cost of equipment repairs (C , in 1000s of dollars) is a function of time:

- if the equipment fails before 1 year, the cost is $C = 0$, because the equipment is insured against failure during this time.
- if the equipment fails between 1 and 10 years, the cost is a linear function of Y ; in particular, $C = 2Y$.
- If the equipment fails after 10 years, the cost is $C = 30$ because new equipment must be purchased.

Find the expected value of C .