GROUND RULES:

- This exam contains 10 questions; each question is worth 10 points. The maximum number of points on this exam is 100.
- Print your name **at the top of this page in the upper right hand corner**.
- This is a closed-book and closed-notes exam. You may use a calculator if you wish. I will provide the discrete/continuous distribution summaries as promised.
- SHOW ALL OF YOUR WORK AND EXPLAIN ALL OF YOUR REASONING!!!
- Any discussion or otherwise inappropriate communication between examinees, as well as the appearance of any unnecessary material, will be dealt with severely.
- You have 3 hours to complete this exam. GOOD LUCK!

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

> *I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.*

1. Suppose that $Y \sim$ exponential(β); i.e., the pdf of Y, for $\beta > 0$, is given by

$$f_Y(y) = \begin{cases} \frac{1}{\beta}e^{-y/\beta}, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Prove that the exponential distribution satisfies the memoryless property; that is, prove

$$P(Y > r + s|Y > r) = P(Y > s),$$

for positive constants r and s.
(b) The exponential distribution is the only continuous distribution that possesses the memoryless property. Name a discrete distribution that also satisfies the memoryless property.

2. A diagnostic test is used to screen subjects for Condition A. The test has *sensitivity* 0.98; that is, the test will declare a subject positive for Condition A with probability 0.98 when the subject is truly positive. The same test has *specificity* 0.90; that is, the test will declare a subject negative for Condition A with probability 0.90 when the subject is truly negative. The proportion of subjects in the population with Condition A is 0.10.
(a) Suppose that a randomly selected subject tests negative. What is the probability s/he is truly positive? *Hint:* Use Bayes Rule.
(b) Suppose that $n = 100$ randomly selected subjects from this population test negative. What is the probability that at least one of the subjects is truly positive? Assume that the $n = 100$ subjects are independent and use your probability p from part (a).

3. For a nationally administered aptitude exam, let X denote a subject's verbal score and let Y denote the subject's quantitative score. Scores have been standardized to fall between 0 and 1; in particular, the joint pdf of (X, Y) is

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{5}(2x+3y), & 0<x<1,\ 0<y<1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the covariance between X and Y.
(b) Are X and Y independent? Explain.

4. Let Y denote the time until failure (in years) of a certain hydraulic component. Suppose that the pdf for Y is

$$f_Y(y) = \begin{cases} 32/(y+4)^3, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Derive the cumulative distribution function (cdf) for Y. Note that there are two cases to consider.
(b) Find the probability that this hydraulic component fails between 2 and 5 years.

5. An insurance company offers four different deductible levels for a homeowner's policy: 0, 1000, 2000, and 5000. It offers three different deductible levels for an automobile policy: 0, 500, and 1000. Let Y_1 denote the homeowner deductible and let Y_2 denote the automobile deductible. For a certain population of customers, the joint probability mass function (pmf) of Y_1 and Y_2 is listed in the table below.

	$y_1 = 0$	$y_1 = 1000$	$y_1 = 2000$	$y_1 = 5000$
$y_2 = 0$	0.04	0.06	0.05	0.03
$y_2 = 500$	0.07	0.10	0.20	0.10
$y_2 = 1000$	0.02	0.03	0.15	0.15

(a) Find the moment generating function of Y_1.
(b) Find the (conditional) mean and variance of Y_2, given $Y_1 = y_1 = 2000$.

6. During regular business hours, customers arrive at a check-out according to a Poisson process with mean $\lambda = 4$ per hour.
(a) In a 2-hour period (during regular business hours), find the probability that exactly 6 customers arrive at the check-out.
(b) Starting at the beginning of regular business hours, find the probability that we will have to wait longer than 30 minutes to see the second customer. NOTE: 30 minutes is 1/2 of an hour.

7. A random variable Y has moment generating function

$$m_Y(t) = \frac{t}{1-t},$$

for $-1 < t < 1$.
(a) Find the mean and variance of Y.
(b) Recall that the skewness associated with Y, denoted by ξ, is given by

$$\xi = E[(Y-\mu)^3]/\sigma^3,$$

where $\mu = E(Y)$ and $\sigma^2 = V(Y)$. Compute ξ.

8. Brazilian scientists have identified a new strain of the H1N1 virus. The genetic sequence of the new strain consists of alterations in the hemagglutinin protein, making it significantly different than the usual H1N1 strain. Public health officials wish to study the population of residents in Rio de Janeiro. Suppose that in this population,

- the probability of catching the usual strain is 0.10
- the probability of catching the new strain is 0.05
- the probability of catching both strains is 0.01.

(a) Find the probability of catching the usual strain or the new strain.
(b) Find the probability of catching the usual strain, given that the new strain is not caught.
(c) Find the probability of catching the new strain, given that at least one strain is caught.

9. An insurance company offers two types of earthquake insurance, Type I and Type II. Let Y_1 and Y_2 denote the claim amounts (in \$10,000s) for the two types, respectively, and assume that the joint pdf of Y_1 and Y_2 is

$$f_{Y_1,Y_2}(y_1,y_2) = \begin{cases} \frac{1}{125} y_1 e^{-(y_1+y_2)/5}, & y_1 > 0,\ y_2 > 0 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Draw a detailed picture of the support of (Y_1, Y_2).
(b) Find the probability that the next claim amount for Type I is larger than twice the next claim amount for Type II. That is, compute $P(Y_1 > 2Y_2)$.
(c) Compute the variance of $U = 2Y_1 - 0.4Y_2$.

10. Suppose that Y is a random variable with pdf

$$f_Y(y) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

The constant $\theta > 0$.
(a) Show that

$$E(Y) = \frac{\theta}{\theta + 1}.$$

(b) Show that the median of Y is

$$\phi_{0.5} = \left(\frac{1}{2}\right)^{1/\theta}.$$

(c) Note that $E(Y)$ and $\phi_{0.5}$ are equal when $\theta = 1$. What is the name of the distribution of Y when $\theta = 1$?