Statistical Multiple Decision Making

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Outline

- Some Motivating Problems.
- Multiple Decision Problems.
- Mathematical Framework (Decision Functions, Losses, Risks).
- **Special Case**: Optimal Choice Between Two Actions.
- Multiple Decision Processes.
- Multiple Decision Size Function.
- Class of FWER-Controlling MDFs.
- Class of FDR-Controlling MDFs.
- An Application to a Microarray Data Set.
- Towards Optimal MDFs.
- Applicability and Some Comparisons.
Some Motivating Questions and Areas of Relevance

- Microarray data analysis: Which genes are relevant?
- Variable selection: Which of many predictors are relevant?
- Survival analysis: Which predictors affects a lifetime variable?
- Reliability: Which components in a system are relevant?
- Epidemiology: Spread of a disease in a geographical area.
- Oil (mineral) exploration: Where to dig?
- Business: Locations of business ventures.
- Sporting Events: Predicting outcomes of NBA playoff games.
A Microarray Data: HeatMap of Gene Expression Levels

First 100 genes out of 41267 genes in a colon cancer study at USC (M Peña’s Lab). Three groups (Control; 9 Days; 2 Weeks) with 6 replicates each.

HeatMap of First 100 Genes
A Typical Variable Selection Problem

- Model.

\[ Y = \beta_0 + \sum_{j=1}^{M} \beta_j X_j + \epsilon \]

- \( M \) is large, but many \( \beta_j \)s are equal to zero.

- Observed Data: For \( j = 1, 2, \ldots, n \),

\[ (Z_j, \delta_j, X_{1j}, X_{2j}, \ldots, X_{Mj}) \]

with

\[ Z_j = \min(Y_j, C_j) \quad \text{and} \quad \delta_j = I\{Y_j \leq C_j\} \]

- Goal: To select the relevant predictor variables.
A Reliability (or Biological Pathways) Problem

- System is composed of components.
- Structure function, $\phi$, relates components to system: series, parallel, series-parallel, etc.
- $M$ potential components that could constitute a system. We do not know which components are relevant nor do we know the structure function.
- **Question**: Given data regarding the states or lifetimes of the system and components, how could we determine which components are relevant for this system?
- Component lifetimes may be censored by system lifetime.
- Highly nonlinear types of relationships.
We would like to discover the value of a parameter
\[ \theta = (\theta_1, \theta_2, \ldots, \theta_M) \in \Theta = \{0, 1\}^M \]

\[ \theta_m = 1 \] means \( m \)th component is relevant; \( \theta_m = 0 \) means \( m \)th component is not relevant.

Want to choose an action
\[ a = (a_1, a_2, \ldots, a_M) \in \mathcal{A} = \{0, 1\}^M \]

\[ a_m = 1 \] means we declare that \( \theta_m = 1 \), called a discovery; \( a_m = 0 \) means we declare that \( \theta_m = 0 \), a non-discovery.
Assessing our Actions: Losses

- Family-wise error indicator:
  \[ L_0(a, \theta) = I \left\{ \sum_{m=1}^{M} a_m (1 - \theta_m) > 0 \right\} \]

- False Discovery Proportion:
  \[ L_1(a, \theta) = \frac{\sum_{m=1}^{M} a_m (1 - \theta_m)}{\max\{\sum_{m=1}^{M} a_m, 1\}} \]

- Missed Discovery Proportion:
  \[ L_2(a, \theta) = \frac{\sum_{m=1}^{M} (1 - a_m) \theta_m}{\max\{\sum_{m=1}^{M} \theta_m, 1\}} \]
If Only We Still Have Paul, the Oracle!

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Statistical Multiple Decision Making
Sadly (or, Gladly), Revert to Being Statisticians!

- Obtain a BIG data (e.g., microarrays, Netflix):
  \[ X \in \mathcal{X} \]

- Probabilistic Structure:
  \[ X \sim P \]

- Marginal Components:
  \[ X_m = z_m(X) \in \mathcal{X}_m \quad \text{and} \quad X_m \sim P_m = Pz_m^{-1} \]

- Parameters of Interest:
  \[ \theta_m = \theta_m(P_m) \]

- Example:
  \[ \theta_m = 1 \iff P_m \in \{ N(\mu, \sigma^2) : \mu \geq 0, \sigma^2 > 0 \} \]
Multiple Decision Functions

- Multiple Decision Function:
  \[ \delta : \mathcal{X} \rightarrow \mathcal{A} \]

- Components:
  \[ \delta = (\delta_1, \delta_2, \ldots, \delta_M) \]
  \[ \delta_m : \mathcal{X} \rightarrow \{0, 1\} \]

- \( \mathcal{D} \): space of multiple decision functions.

- \( \mathcal{M}_0 = \{m : \theta_m = 0\} \) and \( \mathcal{M}_1 = \{m : \theta_m = 1\} \)

- **Structure:** \( \{\delta_m(X) : m \in \mathcal{M}_0\} \) is an independent collection, and is independent of \( \{\delta_m(X) : m \in \mathcal{M}_1\} \).

- \( \{\delta_m(X) : m \in \mathcal{M}_1\} \) need **NOT** be an independent collection.
Given a $\delta \in \mathcal{D}$:

- **Family-Wise Error Rate (FWER):**
  \[ R_0(\delta, P) = E[L_0(\delta(X), \theta(P))] \]

- **False Discovery Rate (FDR):**
  \[ R_1(\delta, P) = E[L_1(\delta(X), \theta(P))] \]

- **Missed Discovery Rate (MDR):**
  \[ R_2(\delta, P) = E[L_2(\delta(X), \theta(P))] \]

Expectations are with respect to $X \sim P$.

**Goal:** Choose $\delta \in \mathcal{D}$ with small risks, whatever $P$ is.
Special Case: A Pair of Choices ($M = 1$)

- $\theta \in \Theta = \{0, 1\}$
- $a \in \mathcal{A} = \{0, 1\}$
- $L_0(a, \theta) = L_1(a, \theta) = aI(\theta = 0)$
- $L_2(a, \theta) = (1 - a)I(\theta = 1)$
- $X \sim P$ with $P \in \{P_0, P_1\}$

- $R_0(\delta, \theta) = R_1(\delta, \theta) = P_0(\delta(X) = 1)I(\theta = 0)$
- $R_2(\delta, \theta) = [1 - P_1(\delta(X) = 1)]I(\theta = 1)$

- Assume $P_0$ and $P_1$ have respective densities:
  
  $f_0(x)$ and $f_1(x)$
Types I and II Errors, Power, and Optimality

- $R_0(\delta, \theta)$: Type I error probability.
- $R_2(\delta, \theta)$: Type II error probability.
- Note
  \[ R_2(\delta, \theta = 1) = 1 - \pi(\delta) \]
  where
  \[ \pi(\delta) = P_1(\delta(X) = 1) = \text{POWER of } \delta. \]
- Desired Goal: Given Type I level $\alpha \in [0, 1]$, find $\delta^*(\cdot; \alpha)$ with
  \[ R_0(\delta^*, \theta) \leq \alpha, \quad \text{for all } \theta, \]
  and
  \[ R_1(\delta^*, \theta) \leq R_1(\delta, \theta), \quad \text{for all } \theta, \]
  for any other $\delta$ with $R_1(\delta, \theta) \leq \alpha, \forall \theta$. 

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Statistical Multiple Decision Making
Neyman-Pearson MP Test $\delta^*_\alpha$

- Neyman and Pearson (1933) obtained the optimal [most powerful] decision function to be of form

$$
\delta^*_\alpha(x) = \begin{cases} 
1 & \text{if } f_1(x) > c(\alpha)f_0(x) \\
\gamma(x) & \text{if } f_1(x) = c(\alpha)f_0(x) \\
0 & \text{if } f_1(x) < c(\alpha)f_0(x)
\end{cases}
$$

where $c(\alpha)$ and $\gamma(\alpha)$ satisfy

$$R_0(\delta^*_\alpha, \theta = 0) = \alpha.$$ 

- Remark: Depends on $\alpha$, hence power depends on $\alpha$.

- Leads to the notion of a decision process.
Concrete Example of a Decision Process

- **Model:** $X = (X_1, X_2, \ldots, X_n) \overset{iid}{\sim} N(\mu, \sigma^2)$.
- **Problem:** Test $H_0 : \mu \leq \mu_0 [\theta = 0]$ vs $H_1 : \mu > \mu_0 [\theta = 1]$
- **Decision Function:** $t$-test of size $\alpha$ given by

$$\delta(X; \alpha) = \mathbf{1}\left\{ \frac{\sqrt{n}(\bar{X} - \mu_0)}{S} \geq t_{n-1;\alpha} \right\}$$

- Decision function depends on the size index $\alpha$.
- **Decision Process:**

$$\Delta = (\delta(\alpha) \equiv \delta(\cdot; \alpha) : \alpha \in [0, 1])$$

- **Size Condition:**

$$\sup\{EP[\delta(X; \alpha)] : \theta(P) = 0\} \leq \alpha$$
Consider a multiple decision problem with $M$ components.  

**Multiple Decision Process:**  

$$\Delta = (\Delta_m : m \in \mathcal{M} = \{1, 2, \ldots, M\})$$

**Decision Process for $m$th Component:**  

$$\Delta_m = (\delta_m(\alpha) : \alpha \in [0, 1])$$

**Example:** $t$-test decision process for each component.

**Usual Approach:** Pick a $\delta_m$ from $\Delta_m$ using the same $\alpha$.

**Common Choices for $\alpha$:** (weak) FWER Threshold of $q$ use:

- **Bonferroni:** $\alpha = q/M$
- **Sidak:** $\alpha = 1 - (1 - q)^{1/M}$
A size function is a function

\[ A : [0, 1] \rightarrow [0, 1] \]

which is continuous, strictly increasing, \( A(0) = 0 \) and \( A(1) \leq 1 \), and possibly differentiable.

- **Bonferroni** size function: \( A(\alpha) = \alpha/M \)
- **Sidak** size function: \( A(\alpha) = 1 - (1 - \alpha)^{1/M} \)

\( \mathcal{G} \): collection of possible size functions.

Given a decision process \( \Delta \) and a size function \( A \), we choose the decision function from \( \Delta \) according to

\[ \delta[A(\alpha)]. \]
For a multiple decision problem with $M$ components, a multiple decision size function is

$$\mathbf{A} = (A_m : m \in \mathcal{M}) \text{ with } A_m \in \mathcal{G}. $$

**Condition:**

$$1 - \prod_{m \in \mathcal{M}} [1 - A_m(\alpha)] \leq \alpha$$

Given a $\Delta = (\Delta_m : m \in \mathcal{M})$ and an $\mathbf{A} = (A_m : m \in \mathcal{M})$, multiple decision function is chosen according to

$$\delta(\alpha) = (\delta_m[A_m(\alpha)] : m \in \mathcal{M})$$

**Weak FWER of $\delta(\alpha)$:**

$$R_0(\delta(\alpha), P) = 1 - \prod [1 - A_m(\alpha)] \leq \alpha$$
Neyman-Pearson Paradigm

- Control Type I error rate; minimize Type II error rate.
- Desired Type I error threshold: \( q \in (0, 1) \)
- **Weak Control:** For \( P \) with \( \theta_m(P) = 0 \) for all \( m \), want a \( \delta \) with

\[
R_0(\delta, P) \leq q \quad \text{or} \quad R_1(\delta, P) \leq q.
\]

- **Strong Control:** Whatever \( P \) is, want a \( \delta \) such that

\[
R_0(\delta, P) \leq q \quad \text{or} \quad R_1(\delta, P) \leq q.
\]

- And, if above Type I error control is achieved, we want to have \( R_2(\delta, P) \) small, if not optimal.
Towards Strong FWER Control

Given a MDP $\Delta = (\Delta_m)$ and MDS $A = (A_m)$, for the chosen $\delta$ at $\alpha$, its FWER is

$$R_0(\delta, P) = E_P \left\{ \prod \left[ \delta_m[A_m(\alpha)] \right] \left[ 1 - \theta_m(P) \right] > 0 \right\}$$

$$= P \left\{ \sum_{\mathcal{M}_0} \delta_m[A_m(\alpha)] > 0 \right\}$$

$$= 1 - \prod_{\mathcal{M}_0} \left[ 1 - A_m(\alpha) \right]$$

$$= 1 - \prod_{\mathcal{M}_0} [1 - A_m(\alpha)]^{1 - \theta_m(P)}$$

**Question:** Given a threshold of $q$, what is the best $\alpha$?
‘Best’ Choice of $\alpha$

- **Oracle Paul**’s Choice:

$$\alpha^\dagger(q; P) = \inf \left\{ \alpha \in [0, 1] : \prod [1 - A_m(\alpha)]^{1 - \theta_m(P)} < 1 - q \right\}$$

- But, $P$ is unknown, hence $\theta_m(P)$ is also unknown. But we could estimate $\theta_m(P)$ by

$$\delta_m[A_m(\alpha) -]$$

- The Oracle’s choice is then estimated by

$$\alpha^\dagger(q) = \inf \left\{ \alpha \in [0, 1] : \prod [1 - A_m(\alpha)]^{1 - \delta_m[A_m(\alpha) -]} < 1 - q \right\}$$
Strong FWER-Controlling MDF

- Chosen Multiple Decision Function:

\[ \delta^\dagger(q) = \left( \delta_m[A_m(\alpha^\dagger(q))] : m \in \mathcal{M} \right) \]

- Theorem

*Given a \( \Delta = (\Delta_m) \) and an \( \mathbf{A} = (A_m) \), the \( \delta^\dagger(q) \) defined above has

\[ R_0(\delta^\dagger(q), P) \leq q \]

whatever \( P \) is. That is, it is an MDF achieving strong FWER control at level \( q \).
**Definition**

The $m$th component of the vector of generalized $P$-value statistic associated with $\Delta$ and $\mathbf{A}$ is

$$
\alpha_m \equiv \alpha_m(\Delta, \mathbf{A}) = \inf\{\alpha \in [0, 1] : \delta_m[A_m(\alpha)] = 1\}
$$

- Smallest size to decide in favor of $\theta_m = 1$ under $(\Delta, \mathbf{A})$.
- Ordered Generalized $P$-Value Statistics:

  $$
  0 \equiv \alpha(0) < \alpha(1) < \alpha(2) < \ldots < \alpha(M) < \alpha(M+1) \equiv 1
  $$

- Observe that for

  $$
  \alpha \in [\alpha(k), \alpha(k+1)) \iff \sum \delta_m[A_m(\alpha)] = k
  $$
Towards FDR Control

Given MDP $\Delta = (\Delta_m)$ and MDS $\mathbf{A} = (A_m)$, the MDF

$$\delta(\alpha) = (\delta_m[A_m(\alpha)] : m \in \mathcal{M})$$

has FDR

$$R_1(\delta(\alpha), P) = E_P \left\{ \frac{\sum \delta_m[A_m(\alpha)](1 - \theta_m(P))}{\sum \delta_m[A_m(\alpha)]} \right\}$$

Observe:

$$E_P \left\{ \sum \delta_m[A_m(\alpha)](1 - \theta_m(P)) \right\} \leq \sum A_m(\alpha)$$
‘Best’ Choice of $\alpha$

- Preceding considerations heuristically suggest the $\alpha$: 
\[
\alpha^*(q) = \sup \left\{ \alpha \in [0, 1] : \sum A_m(\alpha) \leq q \sum \delta_m[A_m(\alpha)] \right\}
\]

- Chosen Multiple Decision Function:
\[
\delta^*(q) = (\delta_m[A_m(\alpha^*(q))] : m \in \mathcal{M})
\]

- Theorem

*Given a pair $(\Delta, \mathbf{A})$, the MDF $\delta^*(q)$ achieves FDR control at level $q$ in that*
\[
R_1(\delta^*(q), P) \leq q.
\]
A class of strong FWER-controlling MDFs at threshold $q$ is:

\[ \mathcal{D}^\dagger = \left\{ \delta^\dagger(q; \Delta, A) : \Delta \in \mathcal{D}, A \in \mathcal{G} \right\} \]

A class of FDR-controlling MDFs at threshold $q$ is:

\[ \mathcal{D}^* = \left\{ \delta^*(q; \Delta, A) : \Delta \in \mathcal{D}, A \in \mathcal{G} \right\} \]

Remark: Sidak’s sequential step-down strong FWER controlling MDF belongs to $\mathcal{D}^\dagger$.

Remark: Benjamini-Hochberg’s step-up FDR controlling MDF belongs to $\mathcal{D}^*$.

Potential Utility: May choose best MDF in $\mathcal{D}^\dagger$ or $\mathcal{D}^*$ wrt the missed discovery rate.
Recalling BH FDR-Controlling MDF

- Let $P_1, P_2, \ldots, P_M$ be the ordinary $P$-values from the $M$ tests.
- Let $P_{(1)} < P_{(2)} < \ldots < P_{(M)}$ be the ordered $P$-values.
- For FDR-threshold equal to $q$, define

$$K = \max \left\{ k \in \{0, 1, 2, \ldots, M\} : P_{(k)} \leq \frac{qk}{M} \right\}.$$

- BH MDF $\delta^{BH}(q) = (\delta^m_{BH} : m \in \mathcal{M})$ has

$$\delta^m_{BH}(X) = I \left\{ P_m \leq P_{(K)} \right\}, \ m \in \mathcal{M}.$$

- Simple and easy-to-implement, but is it the BEST?
Applying BH Procedure to a Two-Group Microarray Data

- Agilent Technology microarray data set from M. Peña’s lab. Jim Ryan of NOAA did the microarray analysis.
- $M = 41267$ genes.
- 2 groups, each group with 5 replicates.
- Applied $t$-test for each gene, using logged expression values. $P$-values obtained.
- Applied Benjamini-Hochberg Procedure with $q = 0.15$ to pick out the significant genes from the $M = 41267$ genes.
- Procedure picked out 2599 significant genes.
- Further analyzed the top (wrt to their $p$-values) 200 genes from these selected genes.
- Performed a cluster analysis on these 200 genes.
Histogram of the $P$-Values from the $t$-Tests

Histogram of data$P.CTFL$

P-Values for CT versus FL
Scatterplot of the Pairwise Gene Means

Significant and Chosen Genes

A Gene
SigGene
Chosen

logCTmean
logFLmean

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Statistical Multiple Decision Making
Pictorial Depiction of Gene Clusters of Top 200 Genes

Clusters in CT vs FL Space

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Statistical Multiple Decision Making
Can We Obtain a Better MDF than BH?

**IDEA:** Given MDP $\Delta = (\Delta_m : m \in \mathcal{M})$, we find the optimal MDS $A^* \equiv A^*(\Delta) \in \mathcal{G}$ achieving smallest MDR

$$R_2[(\Delta \circ A)(\alpha), P_1] = \frac{1}{M} \sum \{1 - \pi_m [A_m(\alpha)]\}.$$

- $\pi_m(\alpha) = \text{POWER of } \delta_m(\alpha)$
- **FWER-controlling MDF:**
  $$\delta^*(q) = \delta^*(q; \Delta, A^*(\Delta))$$
- **FDR-controlling MDF:**
  $$\delta^*(q) = \delta^*(q; \Delta, A^*(\Delta))$$
- Use the best MDP $\Delta$, e.g., MPs; UMPs; UMPUs; UMPIs.
$\textbf{Role of Power or ROC Functions}$

- $P$-value based procedures ignore differences in powers.
- Neyman and Pearson: power germane in search for optimality.
- Power of $m$th Test: $\pi_m(\alpha) = E_{P_m} \{ \delta_m(X; \alpha) \}$
- ROC Function for $m$th Decision Process $\Delta_m$:
  \[ \alpha \mapsto \pi_m(\alpha) \]

- ROC functions in the missed discovery rate.
- Enables exploiting differences in the ROC functions.
- Why Power or ROC Differences? Different effect sizes, decision processes, or dispersion parameters.
- EXCHANGEABILITY: EXCEPTION rather than RULE!
Case with Simple Nulls and Simple Alternatives

- Neyman-Pearson Most Powerful Decision Process for each $m$.
- ROC Functions:
  \[ \alpha \mapsto \pi_m(\alpha) \]
- ROC functions are concave, continuous, and increasing.
- Assume that they are also twice-differentiable.

**Theorem**

*Multiple decision size function* $(\alpha \mapsto A_m(\alpha) : m \in \mathcal{M})$ is **optimal** if it satisfies the $M + 1$ equilibrium conditions

\[
\forall m \in \mathcal{M} : \quad \pi'_m(A_m)(1 - A_m) = \lambda \quad \text{for some } \lambda \in \mathbb{R};
\]

\[
\sum_{\mathcal{M}} \log(1 - A_m) = \log(1 - \alpha).
\]
Example: Optimal Multiple Decision Size Function

- \( M = 2000 \)
- For each \( m \): \( X_m \sim N(\mu_m, \sigma = 1) \)
- Multiple Decision Problem: To test
  \[
  H_{m0} : \mu_m = 0 \quad \text{versus} \quad H_{m1} : \mu_m = \gamma_m.
  \]
- Effect Sizes: \( \gamma_m \) IID \( \sim |N(0, 3)| \)
- For each \( m \), Neyman-Pearson MP decision process.
  \[
  \Delta_m = (\delta_m(\alpha) : \alpha \in [0, 1])
  \]
  \[
  \delta_m(x_m; \alpha) = I\{x_m \geq \Phi^{-1}(1 - \alpha)\}
  \]
- Power or ROC Function for the \( m \)th NP MP Decision Process:
  \[
  \alpha \mapsto \pi_m(\alpha) = 1 - \Phi \left[ \Phi^{-1}(1 - \alpha) - \gamma_m \right]
  \]
Optimal Test Sizes vs Effect Sizes

Density Histogram

Optimal Test Size

Effect Size

Power (Blue=Optimal; Red=Sidak)

Effect Size

Optimal Test Power

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Statistical Multiple Decision Making
Economic Aspect: A Size-Investing Strategy

- Do not invest your size on those where you will not make discoveries (small power) or those that you will certainly make discoveries (high power)!

- Rather, concentrate on those where it is a bit uncertain, since your differential gain in overall discovery rate would be greater!

- Some Wicked Consequences
  - Departmental Merit Systems.
  - Graduate Student Advising.
BH MDF versus $\delta^*(q)$: $q^* = .1$; $M = 20$; 1000 Reps

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BH MDF versus $\delta^*(q)$: $q^* = .1; M = 100; 1000$ Reps

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<td>5.69</td>
<td>3.80</td>
<td>6.14</td>
<td>5.72</td>
</tr>
</tbody>
</table>
Potential Applications and Concluding Remarks

- Microarray data analysis: which genes are important?
- Systems analysis (Biological Pathways?): which components (subsystems of genes) are relevant?
- Variable selection: which predictor variables are important?
- For each gene, component, or predictor variable, apply a decision function to decide whether, say, independence or dependence holds with respect to the response variable.
- Test for Independence: Kendall’s procedure, for example.
- Use MDFs $\delta^+(q)$ or $\delta^*(q)$.
- Issues of determining effect sizes to determine power or ROC functions still need further studies.
- Comparison with other methods, such as those using regularization?
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