<u>Randomized Block Design with Sampling</u>

- **Sometimes we may have more than one observation per treatment-block combination.**

- **<u>Within each block</u>, we have a sample of $n \geq 2$ observations having <u>the same treatment</u>.**

- **Model equation for RBD with sampling:**

$$Y_{ijk} = \mu + \tau_i + \beta_j + \varepsilon_{ij} + \delta_{ijk}$$

$$i = 1, \ldots, t, \quad j = 1, \ldots, b, \quad k = 1, \ldots, n$$

- **ε_{ij} was <u>experimental error</u> → measures variation among the treatment mean responses (across the collection of blocks) [var(ε_{ij}) = σ^2]**

- **δ_{ijk} is <u>sampling error</u> → measures variation among units having the same treatment <u>within the same block</u> [var(δ_{ijk}) = σ_δ^2]**

- **In this situation, we must look carefully at the Expected MS to choose the appropriate denominator for our F-statistic.**

- **Assuming treatment effects are fixed and block effects are random:**

Source	df	Expected(MS)
Treatments	$t-1$	$\sigma_\delta^2 + n\sigma^2 + \frac{nb}{t-1}\sum_i \tau_i^2$
Blocks	$b-1$	$\sigma_\delta^2 + n\sigma^2 + nt\sigma_\beta^2$
Exp. Error	$(t-1)(b-1)$	$\sigma_\delta^2 + n\sigma^2$
Samp. Error	$tb(n-1)$	σ_δ^2
Total	$tbn-1$	

- **Testing for treatment effects:**

Recall H₀: $\tau_1 = \tau_2 = \cdots = \tau_t = 0 \iff H_0: \sum_i \tau_i^2 = 0$

- **If H_0 is true, then which two Mean Squares have the same expected value?** MS(Trts) and MS(Exp.Error)

- **Appropriate test statistic is:**

$$F^* = \frac{MS(Trts)}{MS(Exp.Error)}$$ **Reject H_0 if:** $F^* > F_\alpha[t-1, (t-1)(b-1)]$

- **What is the test statistic for testing H_0: $\sigma_\beta^2 = 0$?**

$$F^* = \frac{MS(Blocks)}{MS(Exp.Error)}$$ **Reject H_0 if:** $F^* > F_\alpha[b-1, (t-1)(b-1)]$

- **What is the test statistic for testing H_0: $\sigma^2 = 0$?**

$$F^* = \frac{MS(Exp.Error)}{MS(Samp.Error)}$$ **Reject H_0 if:** $F^* > F_\alpha[(t-1)(b-1), tb(n-1)]$

Example: **Experiment on stretching ability (Table 10.6, p. 535-536)**

Response = stretching ability of rubber material
Treatments = 7 materials (A, B, C, D, E, F, G)
Blocks = 13 lab sites

- **At each lab, there were *n* = 4 units for each type of material.**

***n* = 4, *t* = 7, *b* = 13 → total of** 364 **observations overall.**

- **Is there a significant difference in mean stretching ability among the seven materials?**

- **We test:** $H_0: \tau_1 = \tau_2 = \cdots = \tau_7 = 0$

(or could write $H_0: \sum_{i=1}^{7} \tau_i^2 = 0$)

F* = $\dfrac{MS(Trts)}{MS(Exp.\ Error)} = \dfrac{44796.35}{330.65} = 135.5$

↑ In SAS, use TEST statement to specify appropriate Error (denominator) term.

Compare to $F_{.05,6,72} \approx 2.22$

Software gives P-value: near 0

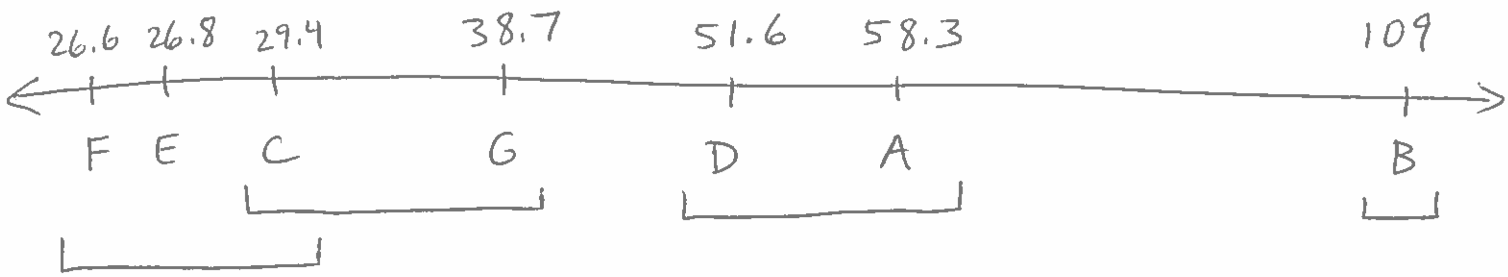
- **Reject H_0 and conclude there is a significant difference in mean stretching ability among the seven materials.**

• **Which of the materials are significantly different in terms of mean stretching ability?**

• **Can use Tukey multiple comparisons procedure (experimentwise error rate $\alpha = 0.05$).**

Results from software:

Sample mean response for each material:



Based on Tukey Procedure, mean responses in the same "grouping" in the graph above are <u>not</u> significantly different.
($\alpha = .05$)

<u>Latin Square Designs</u>

• **Sometimes we may have <u>two</u> blocking factors.**

<u>Example</u>: Suppose we are comparing tire performance across four tire brands (label them A, B, C, D).

• The blocking factors are Car (1, 2, 3, 4) and Tire Position (1, 2, 3, 4).

• If we make each car/position combination a block, we have 16 blocks → we need 64 tires (inefficient and costly!)

• What if we only have 16 tires for the experiment?

A Poor Arrangement:

		Tire Position			
		1	2	3	4
Car	1	A	A	A	A
	2	B	B	B	B
	3	C	C	C	C
	4	D	D	D	D

• Here, the value of car as a blocking factor is lost.

• Each car has only one brand of tire.

- Effect of "brand" would be confounded with the effect of "car".

A Better Arrangement:

		Tire Position			
		1	2	3	4
Car	1	A	B	C	D
	2	B	A	D	C
	3	C	D	A	B
	4	D	C	B	A

- **Now each car gets each brand of tire and each position gets each brand of tire.**

- **This design is called a <u>Latin Square</u>.**

- **Each row and each column contains each treatment <u>once and only once</u>.**

- **A $t \times t$ Latin Square is used for an experiment for t treatments and <u>two</u> blocking factors:**
    - **Row factor with t levels**
    - **Column factor with t levels**

Formal Linear Model for Latin Square:

$$Y_{ijk} = \mu + \rho_i + \gamma_j + \tau_k + \varepsilon_{ijk}$$

$i = 1, \ldots, t$
$j = 1, \ldots, t$
$k = 1, \ldots, t$

ρ_i = effect of row i

γ_j = effect of column j

τ_k = effect of treatment k

ε_{ijk} = random error component

<u>Note</u>: In a Latin Square design, there is assumed to be no interaction!

<u>Example</u> (Table on course web page): Experiment to study the effect of music type on employee productivity

- **Treatments: A = rock & roll, B = country, C = easy listening, D = classical, E = none.**

- **Row factor levels: 5 times of day (9-10, 10-11, 11-12, 1-2, 2-3)**

- **Column factor levels: 5 days of week (Mon, Tue, Wed, Thu, Fri)**