2.4 Discrete Probability Models

Recall: An __________ is the process by which an observation is made.

Defn: A sample point (or outcome) is the particular result of an experiment.

Defn: The sample space (denoted S) of an experiment is the set of all possible sample points.

Defn: A discrete sample space contains a finite or countable number of distinct sample points.

Example 1: Roll a fair die:

$S =$

Example 2: Flip a fair coin until the first "head" and record the number of "tails" that have occurred.

$S =$

Example 3: Randomly select an American household and record the number of TV sets.

$S =$

Defn: An **event** is a collection of sample points (an event is a subset of S)

Note: We typically denote events by capital letters.

- A ________ event corresponds to exactly one sample point.
- A __________ event corresponds to more than one sample point.

Example 1: (Roll die)

Let Event A = {roll an odd number}

Let Event B = {roll a 2}

Example 2: Let event C = "get head on first flip"

C =

Let D = "get head before third flip"

D =

- We usually want to determine the probability of an event of interest.
- To do this, we can often assign probabilities to **each** sample point and then determine which sample point(s) correspond to the event.

3 Probability Axioms (Kolmogorov)

Let S be a sample space and A ($\subset S$) be an event. We assign $P(A)$, the probability of A, such that:

Axiom 1:

Axiom 2:

Axiom 3: If $A_1, A_2, A_3, \ldots$ are pairwise mutually exclusive events in S, then:

Corollary: If $A_1, A_2, \ldots, A_n$ are pairwise mutually exclusive events, then

Proof:

Example 1: Let $A_1 = \{1\}$, $A_2 = \{3\}$, $A_3 = \{5\}$.

$P(A) =$

Probability Rules following from the Axioms

① Complement Rule: $P(A) = 1 - P(\bar{A})$.

Proof:

② $P(\emptyset) = 0$. Proof:

③ If $A \subset B$, then $P(A) \leq P(B)$.

Proof:

Corollary: For any $A \subset S$, $P(A) \leq 1 \ (= P(S))$.

Theorem (Additive Law of Probability):

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof:

Picture:

Corollary: If A and B are mutually exclusive events, then

Note: For 3 events A, B, and C:

$P(A \cup B \cup C) =$

Proof:

- In general, an extended additive law holds for any n events $A_1, A_2, \dots, A_n$.
- In simple cases, we can list the sample points corresponding to an event and determine an appropriate probability for each sample point.
- To obtain the probability of the event A in question, we sum the probabilities of the sample points in A.

Example 1 (a): Toss three coins, each weighted such that "heads" is twice as likely as "tails".

Sample Space:

- When all sample points in S are equally likely, finding P(A) is easier. In this case:

$$P(A) =$$

Example 1(b): Same experiment, except all three coins are fair.

Exercise: If we toss 10 fair coins, what is the probability of obtaining two or more "heads"?