Design of Experiments

● Factorial experiments require a lot of resources

● Sometimes real-world practical considerations require us to design experiments in specialized ways.

● The design of an experiment is the specification of how treatments are assigned to experimental units.

Goal: Gain maximum amount of reliable information using minimum amount of resources.

● Reliability of information is measured by the standard error of an estimate.

● How to decrease standard errors and thereby increase reliability?

● Recall the One-Way ANOVA:

● Experiments we studied used the Completely Randomized Design (CRD).

● The estimate of σ^2 was MSW. This measured the variation among responses for units that were treated alike (measured variation within groups).

● We call this estimating the experimental error variation.

● What if we divide the units into subgroups (called blocks) such that units within each subgroup were similar in some way?

● We would expect the variation in response values among units treated alike within each block to be relatively small.

Randomized Block Design (RBD)

● RBD: A design in which experimental units are divided into subgroups called blocks and treatments are randomly assigned to units within each block.

● Blocks should be chosen so that units within a block are similar in some way.

● Reasons for the variation in our data values:

CRD	**RBD**

- **Benefits of a reduction in experimental error:**
  - **decreases MSW (denominator of F* ratios used in F-tests) → more power to reject null hypotheses**

  - **decreases standard errors of means → shorter CIs for mean responses**

<u>Example 1</u>: Suppose we investigate whether the average math-test scores of students from 8 different majors differ across majors.

- **But … students will be taught by different instructors.**
- **We're not as interested in the instructor effect, but we know it adds another layer of variability.**

Solution:

<u>Example 2</u>: Lab animals of a certain species are given different diets to determine the effect of diet on weight gain.

- **Possible block design:**

Example 3: An industrial experiment is conducted over several days (with a different lab technician each day).
● Possible block design:

Example 4: (Table 10.2 data)
Y = wheat crop yield
experimental units = plots of wheat
treatments = 3 different varieties of wheat
blocks = regions of field

Possible arrangement:

● The data are given in Table 10.2.

● Note: Variety A has the greatest mean yield, but there is a sizable variation among blocks.

● If we had used a CRD, this variation would all be experimental error variance (inflates MSW).

● Analysis as CRD (ignoring blocks):

● But … within each block, Variety A clearly has the greatest yield (RBD will account for this).

Formal Linear Model for RBD

● **This assumes <u>one observation per treatment-block combination</u>.**

Y_{ij} = response value for treatment i in block j
μ = an overall mean response
τ_i = effect of treatment i
β_j = effect of block j
ε_{ij} = random error term

● **Looks similar to two-factor factorial model with one observation per cell.**

<u>Key difference</u>: With RBD, we are not equally interested in both factors.
● **The treatment factor is of primary importance; the blocking factor is included merely to reduce experimental error variance.**

● **With RBD, the block effects are often considered random (not fixed) effects.**

● **This is true if the blocks used are a random sample from a large population of possible blocks.**

● If treatment effects are fixed and block effects are random, the RBD model is called a <u>mixed model</u>.

● In this case, the treatment-block interaction is also random.

● This interaction measures the variation among treatment effects across the various blocks.

● The mean square for interaction is used here as an estimate of the <u>experimental error variance</u> σ^2.

<u>Expected Mean Squares in RBD</u>

Source	**df**	**E(MS)**

● Testing for an effect on the mean response among treatments:

H_0:

● The correct test statistic is apparent based on E(MS):

F* = **Reject H_0 if:**

● Testing for significant variation across blocks:

H_0:

● The correct test statistic is again apparent:

F* = **Reject H_0 if:**

<u>Example</u>: (Wheat data – Table 10.2)

● The ANOVA table formulas are the same as for the two-way ANOVA.

● We use software for the ANOVA table computations.

RBD analysis (Wheat data):

F* =

● We conclude that the mean yields are significantly different for the different varieties of wheat. At α = 0.05, we reject $H_0: \tau_1 = \tau_2 = \tau_3 = 0$.

<u>Note</u> (for testing about blocks):

F* =

● We would also reject $H_0: \sigma_\beta^2 = 0$ and conclude there is significant variation among block effects.

● We can again make pre-planned comparisons using contrasts.

<u>Example</u>: Is Variety A <u>superior</u> to the other two varieties in terms of mean yield?

H_0:

H_a:

Result: