<u>Randomized Block Design with Sampling</u>

● Sometimes we may have more than one observation per treatment-block combination.

● <u>Within each block</u>, we have a sample of $n \geq 2$ observations having <u>the same treatment</u>.

● Model equation for RBD with sampling:

● ε_{ij} was <u>experimental error</u> → measures variation among the treatment mean responses (across the collection of blocks) [var(ε_{ij}) = σ^2]

● δ_{ijk} is <u>sampling error</u> → measures variation among units having the same treatment <u>within the same block</u> [var(δ_{ijk}) = σ_δ^2]

● In this situation, we must look carefully at the Expected MS to choose the appropriate denominator for our F-statistic.

● Assuming treatment effects are fixed and block effects are random:

Source	**df**	**Expected(MS)**

• Testing for treatment effects:

Recall H_0:

• If H_0 is true, then which two Mean Squares have the same expected value?

• Appropriate test statistic is:

F^* = **Reject H_0 if:**

• What is the test statistic for testing H_0: $\sigma_\beta^2 = 0$?

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Example: Experiment on stretching ability (Table 10.6, p. 535-536)

Response = stretching ability of rubber material
Treatments = 7 materials (A, B, C, D, E, F, G)
Blocks = 13 lab sites

● **At each lab, there were $n = 4$ units for each type of material.**

$n = 4$, $t = 7$, $b = 13$ → total of
observations overall.

● **Is there a significant difference in mean stretching ability among the seven materials?**

● **We test:**

F* =

Compare to

Software gives P-value:

● **Reject H_0 and conclude there is a significant difference in mean stretching ability among the seven materials.**

• Which of the materials are significantly different in terms of mean stretching ability?

• Can use Tukey multiple comparisons procedure (experimentwise error rate $\alpha = 0.05$).

Results from software:

Latin Square Designs

• Sometimes we may have two blocking factors.

Example: Suppose we are comparing tire performance across four tire brands (label them A, B, C, D).

● The blocking factors are Car (1, 2, 3, 4) and Tire Position (1, 2, 3, 4).

● If we make each car/position combination a block, we have 16 blocks → we need 64 tires (inefficient and costly!)

● What if we only have 16 tires for the experiment?

A Poor Arrangement:

● Here, the value of car as a blocking factor is lost.

● Each car has only one brand of tire.

A Better Arrangement:

● Now each car gets each brand of tire and each position gets each brand of tire.

● This design is called a Latin Square.

● Each row and each column contains each treatment once and only once.

● A $t \times t$ Latin Square is used for an experiment for t treatments and two blocking factors:

- **Row factor with t levels**
- **Column factor with t levels**

Formal Linear Model for Latin Square:

Note: In a Latin Square design, there is assumed to be no interaction!

Example (Table on course web page): Experiment to study the effect of music type on employee productivity

- **Treatments: A = rock & roll, B = country, C = easy listening, D = classical, E = none.**

- **Row factor levels: 5 times of day (9-10, 10-11, 11-12, 1-2, 2-3)**

- **Column factor levels: 5 days of week (Mon, Tue, Wed, Thu, Fri)**

A 5$\times$5 Latin Square is:

● Each music type appears once on each day and once at each time of day.

● Testing for a significant effect of music type on mean productivity:

F* =

● There is a significant difference in mean productivity among the five music types.

● <u>Note</u>: There is also a significant row effect (time of day) and a significant column effect (day of week).

● Specifically, which music types are significantly different?

● Using Tukey's procedure, we see:

Summary:

● Main advantage of a Latin Square design:

Efficiency – can perform useful tests with relatively few experimental units.

● Main disadvantage: cannot test for any interaction.