STAT 521 – Spring 2019 – HW 6

Note: Show your work wherever possible! All students must do all these problems.

1. A store opens at 8 a.m. and closes at 5 p.m. From 8 until 10 a.m., customers arrive according to a Poisson process with rate of 4 per hour. From 10 until noon, customers arrive according to a Poisson process with rate of 8 per hour. From noon until 2 p.m. the arrival rate increases steadily (linearly) from 8 per hour at noon to 10 per hour at 2 p.m. From 2 p.m. to 5 p.m., the arrival rate drops steadily from 10 per hour at 2 p.m. to 4 per hour at 5 p.m. Determine the probability distribution of the number of customers who enter the store on an entire given day.

2. An insurance company pays out claims on its life insurance policies in accordance with a Poisson process having rate $\lambda = 5$ per week. If the amount of money paid on each policy is exponentially distributed with mean \$2000, what is the mean and variance of the amount of money paid by the insurance company in a four-week span?

3. Consider a birth and death process with birth rates $\lambda_i = (i+1)\lambda, i \geq 0$, where $\lambda = 2$, and death rates $\mu_i = i\mu, i \geq 0$, where $\mu = 1.5$.

   (a) Given the specific values of λ and μ listed above, find the expected time to go from state 0 to state 4.

   (b) Given the specific values of λ and μ listed above, find the expected time to go from state 2 to state 5.

4. Consider two machines, both of which have an exponential lifetime with **mean** $1/\lambda$. There is a single repairman who can service machines with a service time that is exponential with **rate** μ. Let $X(t)$ be the number of broken machines at time t.

   (a) Give the state space of this process $\{X(t)\}$. List all the $\{v_i\}$ values and all the $\{P_{ij}\}$ values.
   (b) Set up the Kolmogorov backward equations. You do not have to solve them.

5. Suppose toddler Todd is going through the "terrible twos" stage of his childhood. When Todd is happy, he remains happy for a length of time that is exponential with a mean of 4 hours. When Todd stops being happy, he next becomes sad with probability 0.3 or else becomes angry with probability 0.7. When Todd is sad, he remains sad for a length of time that is exponential with a mean of 1 hour. When Todd is angry, he remains angry for a length of time that is exponential with a mean of 2 hours. After Todd finishes being either angry or sad, he will fall asleep. He remains asleep for a length of time that is exponential with a mean of 5 hours. After he wakes up, he is happy. Label his status at any given time as follows: 0=asleep, 1=happy, 2=sad, 3=angry.

   (a) Define the state space of the process $\{X(t)\}$, which is Todd's status at time t. List all the $\{v_i\}$ values and all the $\{P_{ij}\}$ values.
   (b) List all the $\{q_{ij}\}$ values (the instantaneous transition rates from one state into another).