STAT 520, Fall 2015: Homework 2

1. Suppose that Z_1 and Z_2 are uncorrelated random variables with $E(Z_1) = E(Z_2) = 0$ and $\text{var}(Z_1) = \text{var}(Z_2) = 1$. Consider the process defined by

$$Y_t = Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t$$

where $e_t \sim \text{iid } \mathcal{N}(0, \sigma_e^2)$ and $\{e_t\}$ is independent of both Z_1 and Z_2.

(a) Prove that $\{Y_t\}$ is stationary.

(b) Let Z_1 and Z_2 be independent $\mathcal{N}(0, 1)$ random variables, and set $\sigma_e^2 = 1$ and $\omega = 0.5$. Use the following R commands to simulate and plot $n = 250$ observations from the $\{Y_t\}$ process:

```
omega=0.5
z=rnorm(2,0,1)
e=rnorm(250,0,1)
Y=rep(0,250)
for(i in 1:250){Y[i]=z[1]*cos(omega*i)+z[2]*sin(omega*i)+e[i]}
plot(Y,ylab="Trigonometric process",xlab="Time",type="o")
```

Describe the appearance of your time series.

(c) Amend the R code above to simulate a realization of the process

$$\widetilde{Y}_t = \beta_0 + \beta_1 t + Z_1 \cos(\omega t) + Z_2 \sin(\omega t) + e_t,$$

where $\beta_0 = 100$, $\beta_1 = 0.05$, $\sigma_e^2 = 1$ and $\omega = 0.5$. For example,

```
Y=rep(0,250)
for (i in 1:250){Y[i]=100+0.05*i+z[1]*cos(omega*i)+z[2]*sin(omega*i)+e[i]}
plot(Y,ylab="Trig process with linear trend",xlab="Time",type="o")
```

Does your $\{\widetilde{Y}_t\}$ process appear to be stationary? What is the effect of adding the linear trend term $\beta_0 + \beta_1 t$ to the model?

(d) Plot the first differences $\{\nabla \widetilde{Y}_t\}$ of your simulated $\{\widetilde{Y}_t\}$ process. For example,

```
dY=diff(Y)
plot(dY,ylab=expression(paste(nabla,tilde(Y)[t])),xlab="Time",type="o")
```

Describe the appearance of this first difference process $\{\nabla \widetilde{Y}_t\}$. In particular, does it appear to be stationary? Are you surprised?

2. Give an example of a process $\{Y_t\}$ that satisfies the following. For each, give $E(Y_t)$ and $\mathrm{var}(Y_t)$. For stationary processes give γ_k; for nonstationary processes give $\mathrm{cov}(Y_s, Y_t)$.

   (a) A process with constant mean but variance that increases with time.

   (b) A stationary process whose autocovariance does not go to zero as time lag goes to infinity.

   (c) A nonstationary process whose autocovariance depends only on time lag.

   (d) A stationary process that has nonzero autocorrelation only at lag $k = 1$.

   (e) A nonstationary process whose first differences are stationary.

Do the following problems in Chapter 2 from Cryer and Chan: 2.7, 2.9(a), 2.10, 2.13, 2.14, and 2.19. In each case $\{e_t\}$ is a zero mean white noise process with $\mathrm{var}(e_t) = \sigma_e^2$.