STAT 520 Final Exam Fall 2015

Throughout $\{e_t\}$ is a zero mean white noise process with $\mathrm{var}(e_t) = \sigma_e^2$. There are 36 problems, each worth 3 points for a possible 108 points out of 100, i.e. 8 points of extra credit. Choose the best answer.

1. Which time series model assumption are you testing when you perform a runs test?
(a) Stationarity.
(b) Independence.
(c) Normality.
(d) Invertibility.

2. Consider an invertible MA(2) process $Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$. Which statement is true?
(a) Its PACF decays in an exponential and possibly sinusoidal manner depending on the roots of the MA characteristic polynomial.
(b) It is always stationary.
(c) Its ACF is nonzero at lags $k = 1$ and $k = 2$ and is equal to zero when $k > 2$.
(d) All of the above.

3. Which function is best suited to determine the orders of a mixed ARMA(p, q) process?
(a) EACF.
(b) PACF.
(c) ACF.
(d) Cross-correlation function.

4. The width of a prediction interval for Y_{t+l} from fitting a nonstationary ARIMA(p, d, q) model, i.e. $d \geq 1$, generally
(a) increases as l increases.
(b) decreases as l increases.
(c) becomes constant for l sufficiently large.
(d) tends to zero as l increases.

5. ARIMA stands for
(a) **A**re you **r**eally **i**gnoring **m**e **a**gain?
(b) **A**uto**r**egressive **i**ntegrated **m**oving **a**verage.
(c) **A** **r**apscallion **is** **m**aking **a**mends.
(d) **A**aaargh! **I'm a** pirate!

6. What are you testing when you use the Shapiro-Wilk test?
(a) Stationarity.
(b) Independence.
(c) Normality.
(d) Invertibility.

7. The `tsdiag` function
(a) computes a diagonal matrix suitable for displaying seasonal time series.
(b) provides standard diagnostics from an `arima` fit.
(c) displays the standardized residuals $\{\hat{e}_t^*\}$ along with Bonferroni-adjusted bands, the ACF of the $\{\hat{e}_t^*\}$, and a series of Ljung-Box test p-values.
(d) Both (b) and (c).

8. Maximum likelihood
(a) is the default used in `arima` after loading the `TSA` package.
(b) maximizes the joint probability distribution of the time series over all possible model parameters, e.g. $p(y_1, \ldots, y_t|\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, \sigma_e^2)$.
(c) estimates are approximately normal in large samples.
(d) All of these.

9. A seasonal MA$(2)_4$ model is written
(a) $Y_t = e_t - \theta_1 e_{t-1}$.
(b) $Y_t = e_t - \Theta_1 e_{t-2} - \Theta_2 e_{t-4} - \Theta_3 e_{t-6} - \Theta_4 e_{t-8}$.
(c) $Y_t = e_t - \Theta_1 e_{t-4} - \Theta_2 e_{t-8}$.
(d) $Y_t = \Phi_1 Y_{t-4} + \Phi_2 Y_{t-8} + e_t$.

10. Here is the R output from fitting an ARMA(1,1) model to the Earthquake data $\{Y_t\}$:

```
> arima(sqrt(earthquake),order=c(1,0,1))
Coefficients:
        ar1     ma1  intercept
     0.8352 -0.4295     4.3591
s.e. 0.0811  0.1277     0.2196
sigma^2 estimated as 0.4294: log likelihood = -98.88, aic = 203.76
```

What is the fitted model?
(a) $Y_t = 4.3591 + 0.8352t - 0.4295Y_{t-1} + X_t, \quad X_t \overset{iid}{\sim} N(0, 0.4294)$.
(b) $(\sqrt{Y_t} - 4.3591) = 0.8352(\sqrt{Y_{t-1}} - 4.3591) + e_t + 0.4295e_{t-1}$.
(c) $Y_t = 0.8352Y_{t-1} + e_t - 0.4295e_{t-1}$.
(d) $(\sqrt{Y_t} - 4.3591) = 0.8352(\sqrt{Y_{t-1}} - 4.3591) + e_t - 0.4295e_{t-1}$.

11. True or False. If $\{\nabla Y_t\}$ is a stationary process, then $\{Y_t\}$ must be stationary.
(a) True
(b) False

12. Consider the seasonal AR$(1)_{12}$ process $Y_t = \Phi Y_{t-12} + e_t$. Which statement is true?
(a) The seasonality is $s = 12$.
(b) The PACF is nonzero only at lag 12.
(c) The ACF decays exponentially (and possibly sinusoidally) at seasonal lags sj, $j = 1, 2, 3, \ldots$ only.
(d) All of these are true.

13. In an analysis, we have determined the following:

- The sample ACF for the series $\{Y_t\}$ has a slow, linear decay.
- The series $\{Y_t\}$ tends to increase over time.
- The first difference process $\{\nabla Y_t\}$ has a sample ACF with a very large, significant spike at lag 1, a smaller significant spike at lag 2, and no spikes elsewhere.

Which model is most consistent with these observations?
(a) MA(1)
(b) ARI(2,1)
(c) AR(2)
(d) IMA(1,2)

14. Suppose that $\{Y_t\}$ is a white noise process with a sample size of $n = 100$. If we performed a simulation to study the sampling variation of r_1, the lag one sample autocorrelation, about 95 percent of our estimates r_1 would fall between
(a) -0.025 and 0.025
(b) -0.05 and 0.05
(c) -0.1 and 0.1
(d) -0.2 and 0.2

15. Consider the nonseasonal process defined by

$$(1 + 0.6B)(1 - B)Y_t = (1 - B + 0.25B^2)e_t.$$

This process is identified by which ARIMA model?
(a) ARIMA(1,1,2)
(b) ARIMA(2,1,1)
(c) ARIMA(2,2,1)
(d) ARIMA(2,1,2)

16. True or False. The process $\{Y_t\}$ identified in Question 15 is stationary.
(a) True
(b) False

17. You have performed the augmented Dickey-Fuller unit test to determine if a series needs to be differenced or not. The p-value for the test of is equal to 0.329. What should you do?
(a) Accept $H_0 : \{Y_t\}$ needs to be differenced.
(b) Reject $H_0 : \{Y_t\}$ needs to be differenced.
(a) Accept $H_0 : \{Y_t\}$ does not need to be differenced.
(b) Reject $H_0 : \{Y_t\}$ does not need to be differenced.

18. Consider an AR(2) model $(1 - \phi_1 B - \phi_2 B^2)Y_t = e_t$. True or False: If the AR(2) characteristic polynomial $\phi(x) = 1 - \phi_1 x - \phi_2 x^2$ has imaginary roots, then this model is not stationary.
(a) True
(b) False

19. In class, we examined the Lake Huron elevation data and decided that an AR(1) model was a good model for these data. Below, the estimated standard errors of the forecast error associated with the next 20 MMSE forecasts under the AR(1) model assumption are given:

```
> round(huron.ar1.predict$se,3)
Start = 2007 End = 2026
 [1] 0.704 0.927 1.063 1.152 1.214 1.258 1.289 1.311 1.328 1.340 1.349 1.355 1.360
[14] 1.363 1.366 1.367 1.369 1.370 1.371 1.371
```

What quantity do these estimated standard errors approach as the lead time l increases?
(a) The overall process mean $\hat{\mu}$.
(b) The white noise process variance $\hat{\sigma}_e^2$.
(c) The AR(1) process standard deviation $\sqrt{\hat{\sigma}_e^2/(1-\hat{\phi}^2)}$.
(d) The white noise process standard deviation $\hat{\sigma}_e$.

20. I have tentatively decided on an ARI(1,1) model for a process with a decreasing linear trend. I now want to use overfitting. Which two models should I fit?
(a) ARI(2,1) and ARIMA(1,1,1)
(b) ARI(1,2) and ARI(2,1)
(c) IMA(1,2) and IMA(2,1)
(d) ARI(1,2) and IMA(2,1)

21. True or False. If $\{Y_t\}$ is a nonstationary process, then $\{\nabla Y_t\}$ must be stationary.
(a) True
(b) False

22. True or False. In an MA(1) process, MMSE forecasts and prediction intervals for lead times $l = 2, 3, 4, \ldots$ will be identical.
(a) True
(b) False

23. I have a process $\{Y_t\}$. The first difference process $\{\nabla Y_t\}$ follows a MA(2) model. What is the appropriate model for $\{Y_t\}$?
(a) MA(1)
(b) ARI(2,1)
(c) IMA(1,2)
(d) ARIMA(0,2,2)

24. Suppose that we have observations from an AR(1) process with $\phi = 0.9$. Which of the following is true?
(a) The scatterplot of Y_t versus Y_{t-1} will display a negative linear trend and the scatterplot of Y_t versus Y_{t-2} will display a negative linear trend.
(b) The scatterplot of Y_t versus Y_{t-1} will display a positive linear trend and the scatterplot of Y_t versus Y_{t-2} will display a positive linear trend.
(c) The scatterplot of Y_t versus Y_{t-1} will display a negative linear trend and the scatterplot of Y_t versus Y_{t-2} will display a random scatter of points.
(d) The scatterplot of Y_t versus Y_{t-1} will display a positive linear trend and the scatterplot of Y_t versus Y_{t-2} will display a random scatter of points.

25. Which method of estimation involves equating sample autocorrelations to population autocorrelations and solving the resulting set of equations for ARMA model parameters? Hint: look in Chapter 7.
(a) Maximum likelihood
(b) Conditional least squares
(c) Method of moments
(d) Bootstrapping

26. In introductory statistics courses, students are taught about the importance of means and standard deviations to measure "center" and "spread" of a distribution. Why aren't these concepts a primary focus in time series?
(a) In most time series models, means and standard deviations are biased estimates.
(b) Means and standard deviations are meaningful only for time series data sets that are nonstationary.
(c) The key feature in time series data is that observations over time are correlated; it is this correlation that we look to incorporate in our models.
(d) Means and standard deviations refer to probability distributions; these distributions are of less importance in time series applications.

27. You have a time series that displays nonconstant variance, i.e. it increases or "fans out" over time. What should you do?
(a) Try `BoxCox.ar` to find a transformation that leads to constant variance.
(b) Difference the data twice, then take the sqare root, i.e. use $\{\sqrt{\nabla^2 Y_t}\}$.
(c) Remove the outlying observations that cause non-constant variance and then impute these missing values.
(d) There is nothing to do. All is lost.

28. In an analysis, we have determined that

- The Dickey-Fuller unit root test for the series $\{Y_t\}$ does not reject a unit root.
- The ACF for the series $\{Y_t\}$ has a very, very slow decay.
- The PACF for the differences $\{\nabla Y_t\}$ has significant spikes at lags 1 and 2 (and is negligible at higher lags).

Which model is most consistent with these observations?
(a) IMA(1,1)
(b) ARI(2,1)
(c) ARIMA(2,2,2)
(d) IMA(2,2)

29. True or false. $\{Y_t\}$ follows an ARIMA(p, d, q) with $d \geq 1$. Then $\{Y_t\}$ is stationary.
(a) True
(b) False

30. True or false. $\{Y_t\}$ follows an ARIMA(p, d, q) with $d \geq 1$. Then $\{\nabla Y_t\}$ is always stationary.
(a) True
(b) False

31. The output from `BoxCox.ar` gives the interval λ in $(-0.3, 0.2)$. The best transformation for $\{Y_t\}$ is
(a) the reciprocal, $1/Y_t$.
(b) the sqare root, $\sqrt{Y_t}$.
(c) the natural log, $\ln Y_t$.
(d) the identity, Y_t.

32. The "integrated" part of ARIMA refers to
(a) A fully automated process for fitting state space models.
(b) Time series is an integral part of statistics.
(c) Differencing is a discrete derivative; integrating, i.e. recovering $\{Y_t\}$ from $\{\nabla Y_t\}$, is the opposite of taking the derivative.
(d) All of these.

33. At the end of the course we attempted to cover seasonal ARIMA$(p, d, q)\times$ARIMA$(P, D, Q)_s$ rather quickly and I suggested simply using
(a) The superposition of sines and cosines, exponential drift, and smoothing splines to model seasonal trends.
(b) Emeril's Original Essence for seasoning hamburgers and fitting time series models.
(c) Spectral analysis in the frequency domain.
(d) Using `auto.arima` in the `forecast` package.

34. An ARIMA(1,1,1) model can be written
(a) $Y_t = \phi Y_{t-1} + e_t - \theta e_{t-1}$.
(b) $Y_t = (1 + \phi)Y_{t-1} - \phi Y_{t-2} + e_t - \theta e_{t-1}$.
(c) $\nabla Y_t = e_t - \theta e_{t-1}$.
(d) $Y_t = \beta_0 + \beta_1 t + X_t$ where X_t is ARMA(1,1).

35. Heuristically, the width of prediction intervals obtained from fitting an ARIMA$(1, 1, 1)$ model increase for Y_{t+l}, $l \geq 1$ because
(a) The width of prediction intervals increase without bound for all ARIMA models.
(b) We are trying to predict a nonstationary time series that has built-in random walk behavior.
(c) MMSE estimates minimize the median difference between the truth and the estimate, increasing tail behavior.
(d) They do not increase; they descrease in width.

36. In the last lecture I fit several models to the thermal temperature data used in the midterm. Which of the following is true?
(a) Assuming a trend, e.g. $Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + X_t$ where X_t is IMA(1,1) can improve fit as measured by the AIC vs. simply assuming Y_t is IMA(1,1).
(b) It is possible to include such trends in the `arima` function and then obtain forecasts.
(c) The type of trend assumed will change the overall direction and width of prediction intervals for Y_{t+l}, $l \geq 1$.
(d) All of these are true.