

Derivation of $\text{var}(\bar{Y})$ for stationary process

$$\begin{aligned}\text{var}(\bar{Y}) &= \frac{1}{n^2} \left(\sum_{i=1}^n Y_i \right)^2 \\ &= \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \text{cov}(Y_i, Y_j) \\ &= \frac{1}{n^2} \left\{ \sum_{i=1}^n \text{var}(Y_i) + 2 \sum_{i < j} \text{cov}(Y_i, Y_j) \right\} \\ &= \frac{1}{n^2} \left\{ n\gamma_0 + 2 \sum_{i=1}^{n-1} (n-i)\gamma_i \right\} \\ &= \frac{1}{n^2} \left\{ n\gamma_0 + 2 \sum_{i=1}^{n-1} (n-i)\rho_i\gamma_0 \right\} \\ &= \frac{1}{n^2} \left\{ n\gamma_0 + \frac{2n}{n} \sum_{i=1}^{n-1} (n-i)\rho_i\gamma_0 \right\} \\ &= \frac{\gamma_0}{n} \left\{ 1 + 2 \sum_{i=1}^{n-1} \frac{n-i}{n} \rho_i \right\}\end{aligned}$$