

STAT 515 - Fall 2003 - Solutions to the Practice Homework

Pg. 170: 3.124 a

$$\begin{aligned} \text{a) } 20! / 10! 6! 4! &= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{4 \cdot 19 \cdot 3 \cdot 17 \cdot 4 \cdot 5 \cdot 7 \cdot 13 \cdot 1 \cdot 11}{2} \\ &= 77597520/2 = 38,798,760 \end{aligned}$$

Pg. 186: 4.22 part a only

Using the standard formulas (page 183 and 184) gives

$$\begin{aligned} \mu = \sum x p(x) &= 10(0.05) + 20(0.20) + 30(0.30) + 40(0.25) + 50(0.10) + 60(0.10) \\ &= 0.5 + 4.0 + 9.0 + 10.0 + 5.0 + 6.0 = 34.5 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \sum (x - \mu)^2 p(x) \\ &= (10 - 34.5)^2 0.05 + (20 - 34.5)^2 0.20 + (30 - 34.5)^2 0.30 + (40 - 34.5)^2 0.25 + (50 - 34.5)^2 0.10 + (60 - 34.5)^2 0.10 \\ &= (-24.5)^2 0.05 + (-14.5)^2 0.20 + (-4.5)^2 0.30 + (5.5)^2 0.25 + (15.5)^2 0.10 + (25.5)^2 0.10 \\ &= 174.75 \end{aligned}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{174.75} = 13.21930$$

Pg. 201: 4.52, also a new part f and g

a) If the psychic is guessing (no ESP) what is the value of p, the probability of a correct decision on each trial? **Only 1 out of 10 boxes is correct, so 1/10.**

b) If the psychic is guessing, what is the expected number of correct decisions in seven trials? **This is a binomial experiment with p=0.1 and n=7, so $\mu = np = 7(0.1) = 0.7$ correct decisions**

c) If the psychic is guessing, what is the probability of no correct decisions in seven trials? **This is a binomial experiment with p=0.1 and n=7, the problem is asking $P(X=0) = \binom{n}{x} p^x (1-p)^{n-x}$
 $= \binom{7}{0} 0.1^0 (1-0.1)^7 = 1 (1) (0.9)^7 = 0.4782969$ You could also answer this one by using just the multiplication rule, and you would end up with the same $(0.9)^7$.**

d) Now suppose the psychic has ESP with p=0.5. What is the probability of no correct decisions in seven trials? **Same as in c, but with a new p. $P(X=0) = \binom{7}{0} 0.5^0 (1-0.5)^7 = 1 (1) (0.5)^7 = 0.0078125$**

e) If the psychic failed on all seven trials, is this evidence against them having ESP? Explain. **Yes. The chance of them getting none correct if they really have ESP of any worth is only 7 out of a 1,000. That isn't very likely!**

f) What is the probability that the psychic would get exactly two correct if they had no ESP? **This one would be really annoying to do out the long way, but is easy if you use the binomial formula.**

$$P(X=2) = \binom{7}{2} 0.1^2 (1-0.1)^5 = 7! / (2!5!) (0.1)^2 (0.9)^5 = 21 (0.1)^2 (0.9)^5 = 0.1240029$$

g) What is the probability that the psychic would get exactly two correct if they had ESP with p=0.5?

$$P(X=2) = \binom{7}{2} 0.5^2 (1-0.5)^5 = 21 (0.5)^2 (0.5)^5 = 0.1640625$$