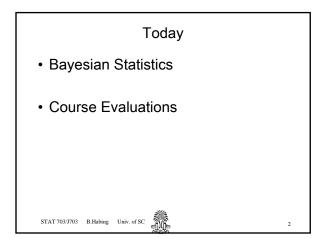
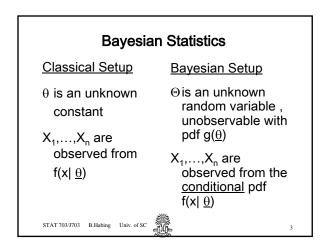


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A Bayesian Statistical Model:

- 1. An observed sample X_1, \dots, X_n , $(X_1, \dots, X_n) = \underline{X} \in X$, the <u>data space</u>.
- An unobserved random vector <u>Θ</u> of parameters, <u>Θ</u>∈Ω, the <u>parameter</u> <u>space</u>.
- 3. The conditional density of $X_1, ..., X_n$, given $\underline{\Theta} = \underline{\theta}$, f(<u>x</u>| $\underline{\theta}$).
- 4. The (marginal) pdf of $\underline{\Theta}$, $g(\underline{\theta})$, the prior distribution of $\underline{\Theta}$.

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Bayes Rule can be written $h(\theta \mid x) = \frac{f(x \mid \theta)g(\theta)}{\int f(x \mid \theta)g(\theta)d\theta}$ Say we knew $f(x \mid \theta)$ and $g(\theta)$ and wanted to find the maximum likelihood estimate of $h(\theta \mid x)$. Why

can we just find the value of θ that maximizes $f(x | \theta) g(\theta)$ and not worry about the bottom integral?

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Hence, inferences are described in terms of the <u>posterior distribution of</u> Θ , the conditional pdf of Θ , given $\underline{X}=(x_1,...,x_n)$, $h(\underline{\Theta}|\underline{x})$. (Interpretation of posterior).

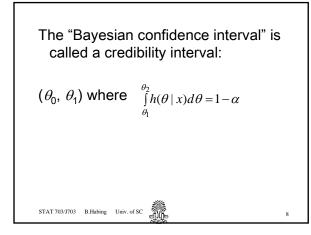
We consider Bayesian <u>point</u> and <u>interval</u> estimators as well as Bayesian <u>tests</u> (<u>decisions</u>). The point estimate is given by a <u>Bayes rule</u>.
$$\label{eq:stample} \begin{split} \underline{Example} & - \text{ Sample } X_1, \dots, X_n \text{ from a normal population whose variance is 1 and mean } \mu \text{ is a value of a random variable } \\ M, \\ X_i | M = \mu \sim N(\mu, 1), \end{split}$$

$$f(x \mid \mu) = \frac{1}{\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2}}, \mu \in R$$

Suppose M ~ N(α , τ^2) as the prior pdf.

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"Bayesian hypothesis testing" depends on a loss function evaluated for competing hypotheses.