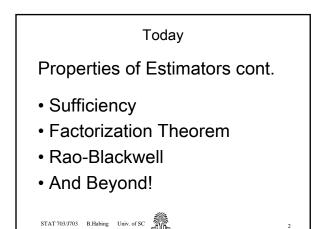


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8.7 Sufficiency One of the key concepts in advanced mathematical statistics is that of sufficiency. Does a statistic summarize all of the information in the data about a parameter, or do we lose something by summarizing.

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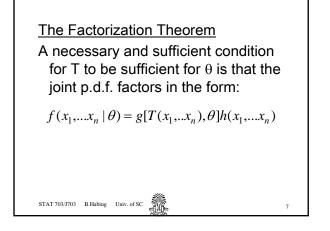
1

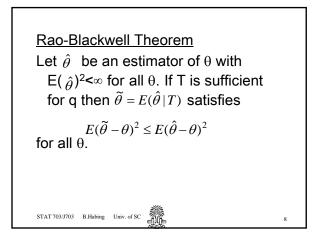
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Example 1: Poisson Distribution

Example 2: Normal Distribution





 $\label{eq:linear} \begin{array}{ll} \underline{\text{Example 3:}} & \text{Consider trying to} \\ \text{estimate } \lambda \text{ for a Poisson distribution} \\ \text{using only } X_1. \end{array}$

Lehmann-Scheffe Theorem Let $\hat{\theta}$ be an <u>unbiased</u> estimator of θ with $E(\hat{\theta})^2 < \infty$ for all θ . If T is <u>complete</u> sufficient for θ then $\tilde{\theta} = E(\hat{\theta}|T)$ is the uniformly minimum variance unbiased estimate (UMVUE) of θ .

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Complete? A statistic T is complete
for θ if the zero function is the only
function that satisfies:
$$E_{\theta}[g(T)] = 0$$
 for all θ

However we have a result similar to the factorization theorem for "exponential families".

$$f(\underline{x} | \underline{\theta}) = \exp[\sum_{i=1}^{k} T_i(\underline{x}) c_i(\underline{\theta}) + d(\underline{\theta}) + S(\underline{x})]$$

where $\underline{\theta} = (\theta_1, \dots, \theta_k)$

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Under appropriate regularity conditions the vector of T's is complete sufficient.

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