STAT 703/J703

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*-Lecture 22-*

Instructor: Brian Habing
Department of Statistics
LeConte 203
Telephone: 803-777-3578
E-mail: habing@stat.sc.edu



Today

Properties of Estimators cont.

- Sufficiency
- Factorization Theorem
- Rao-Blackwell
- And Beyond!



8.7 Sufficiency One of the key concepts in advanced mathematical statistics is that of sufficiency. Does a statistic summarize all of the information in the data about a parameter, or do we lose something by summarizing.

Defn A statistic $T(X_1, \ldots X_n)$ is sufficient for $\theta$ if the conditional distribution of $X_1, \ldots X_n$ given $T=t$ does not depend on $\theta$ for any value of t.

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Example 1: Poisson Distribution

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Example 2: Normal Distribution

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## The Factorization Theorem

A necessary and sufficient condition for T to be sufficient for θ is that the joint p.d.f. factors in the form:

$$f(x_1,\ldots x_n \mid \theta) = g[T(x_1,\ldots x_n),\theta]h(x_1,\ldots x_n)$$



## Rao-Blackwell Theorem

Let $\hat{\theta}$ be an estimator of θ with $E(\hat{\theta})^2<\infty$ for all θ. If T is sufficient for q then $\tilde{\theta} = E(\hat{\theta} \mid T)$ satisfies

$$E(\tilde{\theta}-\theta)^2 \le E(\hat{\theta}-\theta)^2$$

for all θ.



Example 3: Consider trying to estimate λ for a Poisson distribution using only $X_1$.

Lehmann-Scheffe Theorem

Let $\hat{\theta}$ be an unbiased estimator of θ with E($\hat{\theta}$)$^2$<∞ for all θ. If T is complete sufficient for θ then $\tilde{\theta} = E(\hat{\theta} | T)$ is the uniformly minimum variance unbiased estimate (UMVUE) of θ.



Complete? A statistic T is complete for θ if the zero function is the only function that satisfies:

$$E_\theta[g(T)] = 0 \text{ for all } \theta$$



However we have a result similar to the factorization theorem for "exponential families".

$$f(\underline{x} | \underline{\theta}) = \exp[\sum_{i=1}^{k} T_i(\underline{x}) c_i(\underline{\theta}) + d(\underline{\theta}) + S(\underline{x})]$$

where $\underline{\theta} = (\theta_1, \ldots \theta_k)$

Under appropriate regularity conditions the vector of T's is complete sufficient.