STAT 703/J703
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-Lecture 18-

Instructor: Brian Habing
Department of Statistics
LeConte 203
Telephone: 803-777-3578
E-mail: habing@stat.sc.edu

Today

- Homework 5 Solutions
- 9.4: Duality Between Tests and Confidence Intervals

Chapter 9 #6: Develop a likelihood ratio test of H_0: p=0.6 versus H_A: p=0.7 based on n=10 trials.

(Follow the reasoning of Example A of Section 9.3)

Chapter 9 #18: True or False

a) The generalized likelihood ratio statistic L is always less than or equal to 1.

b) If the p-value is 0.03, the corresponding test will reject at the significance level 0.02.

c) If a test rejects at significance level 0.06, then the p-value is less than or equal to 0.06.

d) The p-value of a test is the probability that the null hypothesis is correct.

e) In testing a simple versus simple hypothesis via the likelihood ratio, the *p*-value equals the likelihood ratio.

f) If a chi-square test-statistic with 4 degrees of freedom has a value of 8.5, the *p*-value is less than 0.05.

9.4 The Duality of Confidence Intervals and Hypothesis Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by "inverting" a two-sided test (and vice-versa).

Theorem A, pg. 307: Suppose there is a test of level α for H_0: $\theta = \theta_0$, and let $A(\theta_0)$=acceptance region

Then the set $C=\{\theta: \underline{X} \in A(\theta)\}$ is a $100(1-\alpha)\%$ confidence region for θ.

Theorem B, pg. 307: Let $C(\underline{X})$ be a $100(1-\alpha)\%$ confidence region for θ_0.

Then $A(\theta_0)=\{\underline{X}: \theta_0 \in C(\underline{X})\}$ is an acceptance region for a test of level α for H_0: $\theta = \theta_0$

Example: Consider the random sample from a normal distribution with unknown mean and unknown variance.

Chapter #11.2 Let $X_1,\ldots X_n$ be an independent random sample from a population that is normal with mean μ_x and variance σ^2, and let $Y_1,\ldots Y_m$ be an independent random sample from a population that is normal with mean μ_Y and variance σ^2, such that both samples are independent.

Consider testing about the means.

The likelihood ratio statistic statistic says to reject for large values of

$$1+\frac{mn}{m+n}\left(\frac{(\bar{x}-\bar{y})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2-\sum_{i=1}^{n}(y_i-\bar{y})^2}\right)$$