STAT 703/J703
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-*Lecture 16*-

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Today

- The End!
- 9.5: Contingency Table Example
- 9.4: Duality Between Tests and Confidence Intervals

9.5 Generalized LRT

Observe $X_1,\ldots, X_n$ from $f(x|\theta)$.

Test H_0: $\theta \in \omega_0$ $\omega_0 \subset \Omega$

vs. H_A: $\theta \in \omega_1$ $(\omega_0 \cup \omega_1 = \Omega)$.

$$\Lambda = \frac{\max_{\theta \in \omega_0} \left(lik(\theta)\right)}{\max_{\theta \in \Omega} \left(lik(\theta)\right)} = \frac{\max_{\theta \in \omega_0} \prod_{i=1}^{n} f(x_i \mid \theta)}{\max_{\theta \in \Omega} \prod_{i=1}^{n} f(x_i \mid \theta)}$$

Reject H_0 if $\Lambda \leq \lambda_0$, where λ_0 is

Theorem A pg.310: Under smoothness conditions on the pdf, the null distribution of $-2\ln\Lambda$ has an approximate chi-square distribution with d.f.=$\dim\Omega - \dim\omega_0$ for large n.

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Example: Consider a data set that could have come from a binomial distribution with n=5, but may also have come from a hypergeometric or some other distribution.

X	0	1	2	3	4	5
#obs	9	21	16	10	4	0

Test H_0: X is binomial vs. H_A: it isn't

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So in general we get

$$G^2 = -2\log\Lambda = 2\sum_{i=1}^{n} O_i \log\left(\frac{O_i}{E_i}\right)$$

Page 311-312 demonstrates that this is asymptotically the same as the classic

$$\sum_{i=1}^{n}\frac{(O_i - E_i)^2}{E_i}$$

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Note: You must be careful what estimates you use for the parameters!!!

9.4 The Duality of Confidence Intervals and Hypothesis Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by "inverting" a two-sided test (and vice-versa).

Theorem A, pg. 307: Suppose there is a test of level α for H_0: $\theta = \theta_0$, and let $A(\theta_0)$=acceptance region

Then the set $C=\{\theta: \underline{X} \in A(\theta)\}$ is a $100(1-\alpha)\%$ confidence region for θ.

Theorem B, pg. 307: Let C($\underline{X}$) be a 100(1- α)% confidence region for θ_0.

Then $A(\theta_0)=\{\underline{X}: \theta_0 \in C(\underline{X})\}$ is an acceptance region for a test of level α for H_0: $\theta = \theta_0$

Example cont.: Consider the random sample from a normal distribution with unknown mean and unknown variance.