STAT 703/J703
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-Lecture 15-

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Today

- Homework 4
- 9.5: Generalized Likelihood Ratio Test Revisited
- 9.4: Duality Between Tests and Confidence Intervals

4) A new type of product is supposed to have a mean time until failure of at least 5 hours. A sample of size 8 produced first failure times of 1.6, 4.3, 4.7, 5.8, 6.3, 2.1, 8.5, and 3.2. Determine the appropriate null and alternate hypotheses to determine if the producer should be alerted that they aren't lasting long enough, then set up and perform the best test.

9.5 Generalized Likelihood Ratio Tests Observe $X_1,\ldots, X_n$ from $f(x|\theta)$.

Test H_0: $\theta \in \omega_0 \quad \omega_0 \subset \Omega$

vs. H_A: $\theta \in \omega_1 \quad (\omega_0 \cup \omega_1 = \Omega)$.

Use the generalized LR statistic.

$$\Lambda = \frac{\max\limits_{\theta \in \omega_0} \left(lik(\theta)\right)}{\max\limits_{\theta \in \Omega} \left(lik(\theta)\right)} = \frac{\max\limits_{\theta \in \omega_0} \prod_{i=1}^{n} f(x_i \mid \theta)}{\max\limits_{\theta \in \Omega} \prod_{i=1}^{n} f(x_i \mid \theta)}$$

Reject H_0 if $\Lambda \leq \lambda_0$, where λ_0 is $P(\text{rej. } H_0 | \theta \in \omega_0) = \alpha$.

(Note: If H_0 holds, $\Lambda = 1$. If H_A holds, $\Lambda < 1$, small).

Use this to construct the test, i.e. find rejection regions in terms of simple statistics (similar to N-P lemma).

Example: Consider a random sample from a normal distribution with unknown mean and unknown variance.

Theorem A pg.310: Under smoothness conditions on the pdf, the null distribution of $-2\ln\Lambda$ has an approximate chi-square distribution with d.f.=$\dim\Omega$-$\dim\omega_0$ for large n.

$N(\mu,\sigma^2)$, both unknown and H_0:$\mu=\mu_0$ $\Rightarrow$ df = 2-1 = 1.

9.4 The Duality of Confidence Intervals and Hypothesis Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by "inverting" a two-sided test (and vice-versa).

Theorem A, pg. 307: Suppose there is a test of level α for H_0: $\theta=\theta_0$, and let $A(\theta_0)$=acceptance region

Then the set $C=\{\theta: \underline{X}\in A(\theta)\}$ is a 100(1- α)% confidence region for θ.

Theorem B, pg. 307: Let C(X) be a 100(1- α)% confidence region for θ_0.

Then $A(\theta_0)=\{X: \theta_0 \in C(X)\}$ is an acceptance region for a test of level α for $H_0: \theta = \theta_0$

Example cont.: Again, consider the random sample from a normal distribution with unknown mean and unknown variance.