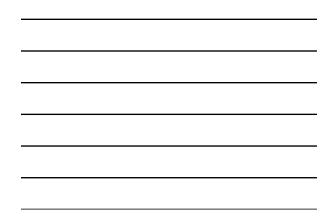


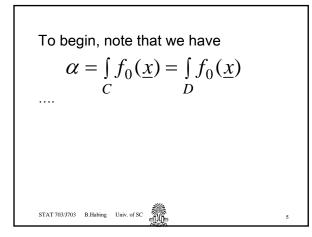
<u>Neyman-Pearson Lemma</u>: If the likelihood ratio test that rejects H_0 in favor of H_A when

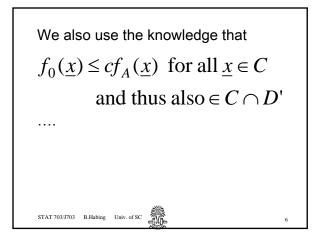
has significance level α ,

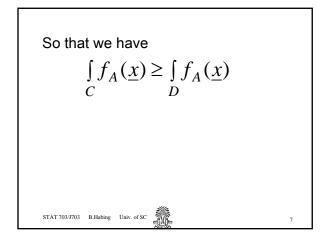
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then <u>any other</u> test having significance level at most α has power <u>less than or</u> <u>equal</u> to the power of the likelihood ratio test. (I.e., the LRT has <u>highest power</u> <u>among tests</u> with significance level α). **Proof:** Let C be the set of possible samples such that $\lambda = \frac{f_0(\underline{x})}{f_A(\underline{x})} \le c$ Let C' be the set of possible samples such that $\lambda = \frac{f_0(\underline{x})}{f_A(\underline{x})} \le c$ Assume there exists some other test with significance level a and it has corresponding regions D and D'











Another Example: The number of down time incidences is supposed to average fewer than 2 per month. Six machines are examined for 2 months each and result in 0, 3, 6, 1, 3, 5 down-time incidences respectively. Conduct a test of the appropriate null and alternate hypotheses to see if you should complain.

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9.5 Generalized Likelihood Ratio Tests

The Neyman-Pearson likelihood ratio test is most powerful for simple vs. simple.

Here, we generalize to composite hypotheses. The generalized LRT is not necessarily optimal, but works well for situations where no optimal test exists.

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Observe X₁, ..., X_n from f(x|
$$\theta$$
).
Test H₀: $\theta \in \omega_0 \quad \omega_0 \subset \Omega$
vs. H_A: $\theta \in \omega_1 \quad (\omega_0 \cup \omega_1 = \Omega)$.
Use the generalized LR statistic.

$$\Lambda = \frac{\max (lik(\theta))}{\max (lik(\theta))} = \frac{\max \prod_{i=1}^n f(x_i | \theta)}{\max \theta \in \Omega \prod_{i=1}^n f(x_i | \theta)}$$
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Reject H₀ if $\Lambda \leq \lambda_0$, where λ_0 is P(rej. H₀| $\theta \in \omega_0$)= α . (<u>Note</u>: If H₀ holds, Λ =1. If H_A holds, Λ <1, small). Use this to <u>construct</u> the test, i.e. find rejection regions in terms of simple statistics (similar to N-P lemma).

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 $\label{eq:condition} \begin{array}{l} \underline{\text{Theorem A}}: \mbox{ Under smoothness} \\ \hline \mbox{ conditions on the pdf, the <u>null} \\ \hline \mbox{ distribution} \mbox{ of } -2ln\Lambda \mbox{ has an} \\ \hline \mbox{ approximate chi-square distribution} \\ \hline \mbox{ with d.f.=dimW-dim} \end{tabular}_0 \mbox{ for large n.} \end{array}$ </u>

 $N(\mu,\sigma^2)$, both unknown \Rightarrow df = 2-1 = 1.

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