STAT 703/J703
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-Lecture 13-

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Today

- 9.3: Neyman-Pearson Lemma
- 9.4: Tests and CIs

Neyman-Pearson Lemma: If the likelihood ratio test that rejects H_0 in favor of H_A when

$$\lambda = \frac{f_0(\underline{x})}{f_A(\underline{x})} \leq c,$$

has significance level α, then any other test having significance level at most α has power less than or equal to the power of the likelihood ratio test. (I.e., the LRT has highest power among tests with significance level α).

Example: Consider a sample of size n from a normal distribution with variance 1.

Test $H_0:\mu=0$ vs. $H_A:\mu=1$ at $\alpha=0.05$.

In some cases we can also show that the test is uniformly most powerful for a composite alternate hypotheses.

This happens if we can show it is most powerful for *every* simple alternate in H_A.

Consider testing
Test $H_0:\mu=0$ vs. $H_A:\mu>0$

and

Test $H_0:\mu=0$ vs. $H_A:\mu\neq0$

Example 2: Consider a binomial distribution with n=8 and unknown p. It is desired to test H_0: p=0.2 versus H_A: p=0.4.

Confidence Intervals and Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by “inverting” a two-sided test (and vice-versa).

Theorem A: Suppose there is a test of level α for H_0: $\theta = \theta_0$, and let $A(\theta_0)$=acceptance region

Then the set $C=\{\theta: \underline{X} \in A(\theta)\}$ is a $100(1-\alpha)\%$ confidence region for θ.

Theorem B: Let C(X) be a 100(1- α)% confidence region for θ_0.

Then $A(\theta_0)=\{\underline{X}: \theta_0 \in C(\underline{X})\}$ is an acceptance region for a test of level α for H_0: $\theta = \theta_0$