STAT 703/J703
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-Lecture 12-

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Today

- Exam 1 Solutions

Neyman-Pearson Lemma: If the likelihood ratio test that rejects H_0 in favor of H_A when

$$\lambda = \frac{f_0(\underset{\sim}{x})}{f_A(\underset{\sim}{x})} \leq c,$$

has significance level α,

then any other test having significance level at most α has power less than or equal to the power of the likelihood ratio test. (I.e., the LRT has highest power among tests with significance level α).

The Rayleigh distribution is used in reliability theory. It is a special case of the Weibull distribution with shape=2 and scale=*q*. It has pdf, mean, and variance

$$f(x\mid\theta)=\frac{x\exp\left(-\frac{x^2}{2\theta^2}\right)}{\theta^2}\text{ for }x>0,\quad \mu=\theta\sqrt{\frac{\pi}{2}},\quad\text{and }\sigma^2=\frac{4-\pi}{2}\theta^2$$

1) Find the formula for the method of moments estimator for θ.

2) Find the formula for the maximum likelihood estimator for θ.

3) Find the asymptotic variance of the mle for θ. [Hint: $E(X^2)=2\theta^2$]

4) Construct an approximate 95% CI for the parameter *q* for the following sample from a population with a Rayleigh distribution.

2.3	3.1	3.4	4.0	4.1
4.5	4.8	6.0	6.1	9.7

5) Use the parametric bootstrap to estimate the standard error for one of the estimators (your choice of method of moments or maximum likelihood) for a sample of size 20 from a Rayleigh distribution with q= 4. [Hint: `rweibull(n, shape=2, scale=sqrt(2)*theta)`]

The pdf, mean, and variance of the lognormal are:

$$f(x \mid m, s) = \frac{1}{xs\sqrt{2\pi}} \exp\left[-\frac{(\ln(x)-m)^2}{2s^2}\right] \text{ for } x > 0,$$

$$\mu = \mathrm{e}^{\frac{1}{2}(2m+s^2)}, \text{ and } \sigma^2 = e^{2m+2s^2} - e^{2m+s^2}$$

6) Find the formula for the method of moments estimators for m and s. [Hint: m^2 has terms that will cancel with s^2 both by addition and division.]

7) Find the formula for the maximum likelihood estimators for m and s.

The pdf, mean, and variance of the F are:

$$f(x \mid m,n) = \frac{\Gamma\left(\frac{n+m}{2}\right) n^{\frac{n}{2}} m^{\frac{m}{2}}}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{m}{2}\right)} \frac{x^{\frac{n}{2}-1}}{(m+nx)^{\frac{n+m}{2}}} \text{ for } x > 0,$$

$$\mu = \frac{n}{n-2}, \text{ and } \sigma^2 = \frac{n^2(2m+2n-4)}{m(n-2)^2(n-4)}$$

Questions 8-10 consider a random sample of size 10 from an F distribution with parameters *m* and *n*.

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0.13   0.18   0.31   0.34   0.44
1.01   1.16   2.77   3.46   5.87
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8) Name a statistical property that both of these estimates share. Then explain which of the two you would rather find the exact formula for if I asked you to for the exam.

9) Use R to find the maximum likelihood estimates for *m* and *n* based on the above sample using 5 and 6 as the initial estimates. [Hint: Notice you can write the log-likelihood in R as: sum(log(df(x,m,n)))]

10) Construct a q-q plot to check if the sample comes from an F distribution with parameters 5 and 6. Does it? If not, briefly describe why not in terms of how the data appears in comparison to the shape of the distribution. (e.g. the data is more skewed left, more skewed right, has heavier tails, etc…)

11) Consider calculating the sample mean for a random sample $x_1, \ldots, x_n$ from a population X with mean μ and variance σ^2. What do we know about the sampling distribution of if we know nothing about the distribution of X? What do we know about the sampling distribution of if X is normally distributed?