STAT 703/J703
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-Lecture 11-

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Today

- Basic Hypothesis Testing Examples
- Neyman-Pearson Lemma

Example 1: Consider a sample of size 1 from a normal distribution with variance 1.

Test $H_0{:}\mu=0$ vs. $H_A{:}\mu=1$ at $\alpha=0.05$.

Example 2: Consider a sample of size 1 from a normal distribution with variance 1.

Test $H_0:\mu\leq 0$ vs. $H_A:\mu>0$ at $\alpha=0.05$.

For a composite test the significance level α is the maximum (supremum) of the probabilities of a Type I error over all the possible alternatives.

9.3 The Neyman-Pearson Lemma:

Typically, there are several possible tests of H_0 vs. H_A for a given level of significance α. How do we select the "best" (in what sense) to use?

"Best" test: A test which has the correct significance level α and is as, or more, powerful ($1-\beta$ is greater) than any other test with the same significance level α.

The Neyman-Pearson theory shows that a "best" test exists for simple H_0 vs. simple H_A and is based on the ratio of the likelikhood functions and on the two hypotheses, i.e. $f_0(\underline{x}) = lik(H_0)$, $f_A(\underline{x}) = lik(H_A)$, where lik(H) is the likelihood function when H is true.

The likelihood ratio, $\lambda = \frac{f_0(\underline{x})}{f_A(\underline{x})}$, gives the "relative plausibilities" of H_0 and H_A.

Reject H_0 if the likelihood ratio λ is small, $\lambda \le c$, where c is chosen to give significance level α.

Neyman-Pearson Lemma: If the likelihood ratio test that rejects H_0 in favor of H_A when

$$\lambda = \frac{f_0(\underline{x})}{f_A(\underline{x})} \le c,$$

has significance level α, then any other test having significance level at most α has power less than or equal to the power of the likelihood ratio test. (I.e., the LRT has highest power among tests with significance level α).

Example: Consider a sample of size n from a normal distribution with variance 1.

Test H_0:μ=0 vs. H_A:μ=1 at α=0.05.

In some cases we can also show that the test is uniformly most powerful for a composite alternate hypotheses.

This happens if we can show it is most powerful for *every* simple alternate in H_A.

Consider testing
Test H_0:μ=0 vs. H_A:μ>0

and

Test H_0:μ=0 vs. H_A:$\mu \neq$0