STAT 703/J703
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-Lecture 10-

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Today

- Basics of Hypothesis Testing Continued

9.2 Neyman-Pearson Paradigm

Let $\underline{X} = (X_1, \ldots, X_n)$ denote a sample from population $f(x|\theta)$. Decide on H_0 vs. H_A based on the sample.

A decision on whether or not to reject H_0 in favor of H_A is made on the basis of a statistic

$$T=T(\underline{X})=T(X_1, \ldots, X_n).$$

The set of values of T for which H_0 is accepted is called the acceptance region and the set of values of T for which H_0 is rejected is the rejection region of the test.

Two kinds of error may occur:

1. H_0 is rejected when it is true: Type I error.

P(type I error) = α
= P(T $\in$ rejection region | H_0 true).

If H_0 is simple, α is called the significance level of the test.

2. H_0 is accepted when it is false: Type II error.

P(type II error) = β
= P(T in acceptance region | H_0 false)

If H_A is composite, β depends on which member of H_A is the true pdf.

Power of the test =P(H_0 is rejected when false)

= 1 - P(H_0 is accepted | H_0 false)
= 1 - β.

Ideally, we would want α=β=0, but this not possible since the decision is based on data.

Example:

Consider testing
H_0: p=0.5
vs. H_A: p=0.6
for a binomial sample of size n=10.

P-value The p-value is the probability of observing a test statistic at least as extreme as the one observed if the null hypothesis is true.

The null hypothesis is rejected when p-value is $\leq \alpha$. It is the smallest α for which H_0 would be rejected.

Example 2:

Consider testing

H_0: p=0.5

vs. H_A: p>0.5

for a binomial sample of size n=10.

For a composite test the significance level α is the maximum (supremum) of the probabilities of a Type I error over all the possible alternatives.