

STAT 703/J703  
February 3<sup>rd</sup>, 2005

-Lecture 8-

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Today

- Information Function (cont.)
- Asymptotic Normality of the MLE

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8.5.2 cont. - Information function

$$I(\theta) = E \left[ \left( \frac{\partial}{\partial \theta} \log f(X | \theta) \right)^2 \right]$$
$$= -E \left[ \frac{\partial^2}{\partial \theta^2} \log f(X | \theta) \right]$$

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Example: Consider a random sample from a Poisson distribution.

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What if there is more than one parameter?

In this case you get an information matrix:

$$I(\theta) = E \left[ \left( \frac{\partial}{\partial \theta_i} \log f(X | \theta) \right) \left( \frac{\partial}{\partial \theta_j} \log f(X | \theta) \right) \right]$$
$$= -E \left[ \frac{\partial^2}{\partial \theta_i \partial \theta_j} \log f(X | \theta) \right]$$

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Theorem B: Under appropriate regularity conditions the MLE is asymptotically normal with mean  $\theta$  and variance  $\frac{1}{nI(\theta)}$ .

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Sketch of proof: Consider the Taylor series expansion:

$$L'(\hat{\theta}) \approx L'(\theta) + (\hat{\theta} - \theta)L''(\theta)$$



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Example 1: Consider a random sample from a Poisson distribution.



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Example 2: Recall the multinomial example from section 8.5.1.



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