STAT 703/J703
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-Lecture 7-

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Today

- Consistency of the MLE
- Information Function
- Asymptotic Normality of the MLE

8.5.2 – Large Sample Properties

Theorem A: The MLE is Consistent (under appropriate regularity conditions)

Sketch of Proof: Consider maximizing $\frac{1}{n}L(\theta)=\frac{1}{n}\sum_{i=1}^{n}\log f(X_i\,|\,\theta)$

Information function

$$I(\theta) = E\left[\left(\frac{\partial}{\partial\theta}\log f(X\,|\,\theta)\right)^2\right]$$

$$= -E\left[\frac{\partial^2}{\partial\theta^2}\log f(X\,|\,\theta)\right]$$

Example: Consider a random sample of size n from a normal distribution with unknown mean θ and known variance σ^2.

What if there is more than one parameter?

In this case you get an information matrix:

$$I(\theta) = E\left[\left(\frac{\partial}{\partial\theta_i}\log f(X\,|\,\theta)\right)\left(\frac{\partial}{\partial\theta_j}\log f(X\,|\,\theta)\right)\right]$$

$$= -E\left[\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log f(X\,|\,\theta)\right]$$

Theorem B: Under appropriate regularity conditions the MLE is asymptotically normal with mean θ and variance $\frac{1}{nI(\theta)}$.

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Sketch of proof: Consider the Taylor series expansion:

$$L'(\hat{\theta}) \approx L'(\hat{\theta}) + (\hat{\theta} - \theta)L''(\hat{\theta})$$

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Example: Recall the multinomial example from section 8.5.1.

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