STAT 703/J703
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-Lecture 6-

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Today

- Homework Solutions
- Logistic Regression MLE example
- Sections 8.5.2-8.5.4: Large Sample Properties of the MLE

28a)

5.3299	4.2537	3.1502	3.7032
1.6070	6.3923	3.1181	6.5941
3.5281	4.7433	0.1077	1.5977
5.4920	1.7220	4.1547	2.2799

What would you guess the mean and variance of the underlying normal distribution to be? (and why?)

44) X_1, … X_n are iid from a distribution with density

$$f(x\,|\,\theta) = (\theta+1)x^{\theta}, \qquad 0 \le x \le 1$$

a) Find the MOM estimate.

b) Find the MLE

MLE example 2 – Logistic Regression

In linear regression we try to predict one continuous variable from another.

Under a few basic assumptions this results in fairly easy parameter estimation and use of t and F distributions.

Consider the case of attempting to predict a 0,1 variable from a continuous variable.

$$P[Y_i = 1 \mid x_i] = \frac{1}{1+e^{-(\alpha+\beta x_i)}}$$

8.5.2 – Large Sample Properties

Theorem A: The MLE is Consistent (under appropriate regularity conditions)

Sketch of Proof: Consider maximizing $\frac{1}{n}L(\theta) = \frac{1}{n}\sum_{i=1}^{n}\log f(X_i \mid \theta)$

Information function

$$I(\theta) = E\left[\frac{\partial}{\partial\theta}\log f(X \mid \theta)\right]^2$$

$$= -E\left[\frac{\partial^2}{\partial\theta^2}\log f(X \mid \theta)\right]^2$$

Theorem B: Under appropriate regularity conditions the MLE is asymptotically normal with mean θ and variance $\frac{1}{nI(\theta)}$.

Sketch of proof: Consider the Taylor series expansion:

$$L'(\hat{\theta}) \approx L'(\hat{\theta}) + (\hat{\theta} - \theta)L''(\hat{\theta})$$