STAT 703/J703
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-Lecture 5-

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Today

- Homework Solutions
- Two additional MLE examples
- Sections 8.5.2-8.5.4: Large Sample Properties of the MLE

1) Find the formula for the method of moment estimator for a geometric random variable.

Note, for a beta distribution:

$$\hat{\alpha} = \left(\frac{\bar{x}}{\hat{\sigma}^2}\right)\left(\bar{x} - \bar{x}^2 - \hat{\sigma}^2\right) \qquad \hat{\beta} = \left(\frac{1-\bar{x}}{\hat{\sigma}^2}\right)\left(\bar{x} - \bar{x}^2 - \hat{\sigma}^2\right)$$

2) Use method of moments to estimate the beta distribution α and β parameters for column c of the data set `itests.txt`

3) Construct a q-q plot to verify if column c of the itests data seems to be from a beta distribution with the parameter estimates you found in (2)

4) Use the parametric bootstrap to estimate the bias and standard error of the method of moments estimators you found in (2).

MLE Example 1 (Section 8.5.1)

Consider a multinomial experiment, where each of the n observations has probability p_i of falling in cell i=1…m.

$$f(x_1,\dots x_m \mid p_1\dots, p_m) = \frac{n!}{\prod_{i=1}^{m} x_i!} \prod_{i=1}^{m} p_i^{x_i}$$

Often there is some restriction on the probabilities however. Consider the case in genetics where a gene occurs with frequence θ in the population.

Say we observe 342 ++, 500 +- and 187 --. What is our estimate of θ?

MLE example 2 – Logistic Regression

In linear regression we try to predict one continuous variable from another.

Under a few basic assumptions this results in fairly easy parameter estimation and use of t and F distributions.

Consider the case of attempting to predict a 0,1 variable from a continuous variable.

$$P[Y_i = 1 | x_i] = \frac{1}{1 + e^{-(\alpha + \beta x_i)}}$$

8.5.2 – Large Sample Properties

Theorem A: The MLE is Consistent (under appropriate regularity conditions)

Information function

$$I(\theta) = E\left[\frac{\partial}{\partial\theta}\log f(X\mid\theta)\right]^2$$

$$= -E\left[\frac{\partial^2}{\partial\theta^2}\log f(X\mid\theta)\right]^2$$

Theorem B: Under appropriate regularity conditions the MLE is asymptotically normal with mean θ and variance $\frac{1}{nI(\theta)}$.