

STAT 703/J703

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-Lecture 21-

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Today

Some Properties of Estimators

- Efficiency
- Cramer-Rao Inequality
- Sufficiency

8.7 Comparing Estimates and Tests

One of the standard tools for evaluating an estimate is the mean squared error:

$$\begin{aligned}MSE(\hat{\theta}) &= E(\hat{\theta} - \theta_0)^2 \\ &= Var(\hat{\theta}) + [E(\hat{\theta}) - \theta_0]^2\end{aligned}$$

If two estimators are unbiased, then the efficiency of $\hat{\theta}$ relative to $\tilde{\theta}$ is

$$eff(\hat{\theta}, \tilde{\theta}) = \frac{\text{var}(\tilde{\theta})}{\text{var}(\hat{\theta})}$$

For two tests T_1 and T_2 of the same H_0 and H_A with the same α -level, the relative efficiency of T_2 to T_1 is the ratio n_2/n_1 required so that they have the same power.

The asymptotic relative efficiency of the MWW to the t -test is:

- 0.955 if the populations are normal
 - 1.0 if the populations are uniform
 - 1.5 if the pops. are double-exp.
 - 0.864 to infinity in general
- assuming the populations differ only by location.

The asymptotic relative efficiency of the MWW to the median test is:

- 1.5 if the populations are normal
- 3.0 if the populations are uniform
- 0.75 if the pops. are double-exp. assuming the populations differ only by location.

Cramer-Rao Inequality

Let X_1, \dots, X_n be i.i.d. with density $f(x|\theta)$, and T be an unbiased estimate of θ . Then under appropriate smoothness assumptions on f

$$\text{Var}(T) \geq \frac{1}{nI(\theta)}$$

Recall that under smoothness conditions that the $1/n I(\theta)$ is the asymptotic variance of the MLE!

So, why isn't the MLE always best?

8.8 Sufficiency One of the key concepts in advanced mathematical statistics is that of sufficiency. Does a statistic summarize all of the information in the data about a parameter, or do we lose something by summarizing.

Defn A statistic $T(X_1, \dots, X_n)$ is sufficient for θ if the conditional distribution of X_1, \dots, X_n given $T=t$ does not depend on θ for any value of t .

Example: Consider a Poisson Distribution with parameter λ and a sample size of 2.

A) Consider $T=X_1 + X_2$

B) Consider $T=X_1 + 2X_2$
