Practice Problems for MoM and Maximum Likelihood

1) Consider a random sample $x_1, \ldots x_n$ from a distribution with pdf

$$f(x) = (\theta+1)(1-x)^{\theta} \quad \text{for } 0<x<1$$

a) Find the MoM estimator for θ.

b) Find the MLE estimator for θ.

2) In many cases a process will have a minimum value > 0 so that using a distribution like the exponential, chi-squared, F, or gamma doesn't make much sense. In this case the distribution can be shifted to the right. Consider the shifted exponential with pdf

$$f(x) = \frac{1}{\theta} e^{-(x-a)/\theta} \quad \text{for } x>a$$

and the data set 16.2,12.4, 6.0, 8.4, 6.8, 9.1, 6.6, 6.0, 10.7, 5.8.

a) Show that the mean of the shifted exponential is $\theta+a$ and the variance is θ^2.

b) Find the form of the MoM estimators for a and θ.

c) Find the form of the MLE for a and θ. (Note that when you take the derivative with respect to a that it can never equal zero, so the maximum must happen at one of the end-points. Also note that the bigger a is the bigger the log-likelihood is. Based on what you know about the pdf, what is the biggest value that a can have?)

3) Consider the Cauchy distribution centered at θ, that has pdf

$$f(x) = \frac{1}{\pi(1+(x-\theta)^2)} \quad \text{for } -\infty<x<\infty$$

a) Why can't there be a MoM estimator for θ?

b) What formula must the MLE satisfy?

c) Use R to find the estimate of θ based on the sample –0.6, 4.2, 1.1, -4.3, -10.3, 1.6, 4.8, 30.9, 0.4, 1.5.