Homework 12 – Problem #2

Let X and Y be independent uniform $[0,1]$ random variables.

Consider the (seemingly ugly) transformations:

$$U = \sqrt{-2\ln(X)}\cos(2\pi Y)$$
$$V = \sqrt{-2\ln(X)}\sin(2\pi Y)$$

a) Demonstrate that:

$$X = \exp\left(-\frac{U^2+V^2}{2}\right)$$
$$Y = \frac{1}{2\pi}\arctan\frac{V}{U}$$

b) Use the transformation of variable formula to find the joint distribution of U and V, and remember to specify where it is defined.

c) Identify the joint distribution by name.

Hints - Some relevant reminders from Trigonometry and Calculus

$$\sin^2\theta + \cos^2\theta = 1$$
$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\frac{d}{du}\arctan u = \frac{1}{1+u^2}du$$

and remember the chain rule!!