STAT 702/J702

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-Lecture 27-

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Today

- An important Lemma
- χ^2 distribution and the variance
- *t* distribution and the mean
- *F* distribution and variances

<u>Lemma:</u> If $X_1, \ldots, X_n$ are i.i.d. $N(\mu,\sigma^2)$ then $\bar{X}$ and $(X_1-\bar{X}, X_2-\bar{X}, \ldots, X_n-\bar{X})$ are independent.

χ^2 distribution and the variance

The $\chi^2_{df=v}$ is a special case of the gamma distribution where

$\alpha = v/2$ and $\lambda = 1/2$

and v is the degrees of freedom.

Recall that:

If Z~N(0,1) then $Z^2 \sim \chi^2_{df=1}$

If $X_1, \ldots X_k$ are independent χ^2 with df = $v_1, \ldots v_k$ then $\Sigma X_i \sim \chi^2_{df=\Sigma vi}$

<u>Theorem:</u> If $X_1, \ldots, X_n$ are i.i.d. $N(\mu,\sigma^2)$ then $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{df=n-1}$

Definition: If Z~N(0,1) and X ~ $\chi^2_{df=v}$ are independent then

$$\frac{Z}{\sqrt{\frac{X}{v}}} \sim t_{df=v}$$

Theorem: If $X_1, \ldots, X_n$ are i.i.d.

$N(\mu,\sigma^2)$ then $\frac{\overline{X}-\mu}{s/\sqrt{n}} \sim t_{df=n-1}$

Definition: If U~ $\chi^2_{df=m}$ and V~ $\chi^2_{df=n}$ independent then (U/m)/(V/n) ~ $F_{df=m,n}$

Theorem: If $X_1, \ldots, X_{nx}$ are i.i.d. $N(\mu_X,\sigma_X^2)$ and $Y_1, \ldots, Y_{ny}$ are i.i.d. $N(\mu_Y,\sigma_Y^2)$ then

$$\frac{s_X^2 / \sigma_X^2}{s_Y^2 / \sigma_Y^2} \sim F_{df=n_X-1,n_Y-1}$$