STAT 702/J702

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-Lecture 25-

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Today

- Homework Solutions
- Chebyshev and LLN
- CLT

Ch 1 # 73) A system has n independent units, each of which fails with probability p. The system fails only if k or more of the units fail. What is the probability that the system fails?

Chapter 3 #55) Suppose that a system's components are connected in series and have lifetimes that are independent exponential random variables with parameters λ_i. Shat the the lifetime of the system is exponential with parameter $\Sigma\lambda_i$.

Chapter 4 #62) Show that $E[Var(Y|X)] \leq Var(Y)$

Chebyshev's Inequality

$$P(|X-\mu|>k\sigma)=$$

$$=\int_{|x-\mu|>k\sigma} f(x)dx \quad \leq \int_{|x-\mu|>k\sigma} \frac{|x-\mu|}{k\sigma} f(x)dx$$

$$\leq \int_{|x-\mu|>k\sigma} \frac{(x-\mu)^2}{k^2\sigma^2} f(x)dx$$

$$\leq \int_{-\infty}^{\infty} \frac{(x-\mu)^2}{k^2\sigma^2} f(x)dx$$

$$= \frac{1}{k^2\sigma^2} \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$$

$$= \frac{\sigma^2}{k^2\sigma^2} = \frac{1}{k^2}$$

So:

$$P(|X-\mu| > k\sigma) \leq \frac{1}{k^2}$$

Weak Law of Large Numbers (Sect. 5.2)

Chebyshev's inequality can be used to prove the weak law of large numbers:

If $X_1,\ldots.X_i,\ldots$ is a sequence of independent random variables with mean μ and variance σ^2, then

$$P(|\overline{X}_n - \mu| > \varepsilon) \to 0$$

as $n \to \infty$.

This is an example of Convergence in Probability.

The probability of being far away from the limit goes to zero as $n \to \infty$.

Unfortunately the LLN doesn't give us a good feeling for how close $\overline{X}$ should be to μ.

The central limit theorem provides some guidance in this respect. The most common version of the CLT says that:

Let X_1, X_2, … be a sequence of independent identically distributed random variables with mean μ and variance σ^2. Then

$$\lim_{n \to \infty} P\left(\frac{\sum_{i=1}^{n} X_i - \mu}{\sigma / \sqrt{n}} \leq x \right) = \Phi(x)$$

$$-\infty < x < \infty$$