STAT 702/J702

November 16th, 2004

-Lecture 24-

Instructor: Brian Habing
Department of Statistics
Telephone: 803-777-3578
E-mail: habing@stat.sc.edu

Today

- Homework Solutions
- Application 3: Random Sums
- Applicaton 4: Interpreting Variances

Ch 4 # 75) Find the m.g.f. of a Bernoulli random variable, and use it to find the mean, variance, and third moment.

Ch 4 # 76) Use the result of problem 75 to find the m.g.f. of a binomial random variable and its mean and variance.

Random Sums

An insurance company receives N independent claims $X_1, \ldots X_N$ in a given time period. Where N is also a random variable (independent of the X_i).

What are the mean and variance of

$$T = \sum_{i=1}^{N} X_i$$

This would be much easier to work with if we could condition on N and consider $T|N$.

$$E(T \mid N = n) = E\left(\sum_{i=1}^{N} X_i \mid N = n\right)$$
$$= E\left(\sum_{i=1}^{n} X_i\right)$$
$$= \sum_{i=1}^{n} E(X_i) = nE(X)$$

But we somehow need to take the expectation over N as well.

The general result is

$E(Y)=E_X[E_{Y|X}(Y|X)]$

A similar result is

$Var(Y)=Var[E(Y|X)]+E[Var(Y|X)]$

Interpreting Variances

If a population is normal (or nearly normal) then the variance is fairly easy to interpret.

What if the population is not normal?

Chebyshev's Inequality relates the probability of being within a certain range of the mean to the variance for any distribution.

Law of Large Numbers (Section 5.2)

Chebyshev's inequality can be used to prove the law of large numbers:

If $X_1,\ldots.X_i,\ldots$ is a sequence of independent random variables with mean μ and variance σ^2, then

$$P(|\overline{X}_n - \mu| > \varepsilon) \to 0$$

as $n \to \infty$.