STAT 702/J702

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-Lecture 21-

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Today

- Exam 2
- Moment Generating Functions continued

Exam 2 #1) Let X and Y be independent random variables where X is exponential with λ=2 and Y is normal with μ=4 and σ=6. Find E(X+Y) and Var(X+Y).

2) Find k such that $f(x) = kx(1-x)$ on $0<x<1$ (and is 0 elsewhere) is a p.d.f.

3) Let the random variable X have c.d.f. $F(x) = \frac{1}{2}(1 + x^3)$ on $-1 \leq x \leq 1$ (and 0 otherwise). Find E(X) and Var(X).

4) Leaks due to manufacturing defects occur in a brand of hose at a rate of approximately 1 per 500 feet. Name an appropriate distribution and estimate the probability that the first defect will be found in the first 100 feet.

5) Evaluate $\int_0^{\infty} x^2 e^{-\pi x}\,dx$

6) Let X have a uniform distribution on $(-\pi/2, \pi/2)$. Find the c.d.f. and p.d.f. of Y=tanX.

7) Let $f_{XY}(x,y) = \frac{1}{4} + \frac{xy}{16}$ on $-1<x<1, -1<y<1$ (and be 0 otherwise). Find the conditional distributions of X|Y and Y|X. Also, are X and Y independent?

8) Let X and Y be independent chi-square random variables with 1 degree of freedom. (The p.d.f. is on page 59.) Derive the p.d.f. of Z=X/Y.

9) In class we showed how to get general formula's for the p.d.f.'s of Z=X/Y and Z=X+Y. Show that if X and Y have joint p.d.f. $f_{XY}(x,y)$ and Z=XY that $f_Z(z)=\int_{-\infty}^{\infty} f_{XY}\left(x,\frac{z}{x}\right)\frac{1}{|x|}dx$.

10) Let X and Y have joint p.d.f. $f_{XY}(x,y)$ = 1 + (1-2x)(1-2y) on $0<x<1, 0<y<1$ (and be 0 elsewhere). Find the joint p.d.f. of U=X+Y and V=X+2Y.

11) Let $X_1,\ldots X_{11.}$ be independent exponential random variables with parameter $\lambda=1$. Find the p.d.f. for the median $X_{(6)}$.

4.5 – Moment Generating Functions

The moment-generating function (mgf) of X is $M(t)=E(e^{tX})$

$$M(t)=\sum_x e^{tX}\, p(x)$$

$$M(t)=\int_{-\infty}^{\infty} e^{tX}\, f(x)dx$$

Properties of m.g.f.'s

a) If the m.g.f. exists on an interval around zero then $M^{(k)}(0)=E(X^k)$

b)The m.g.f. uniquely determines the p.d.f.

c) If $Y=a+bX$ then $M_Y(t)=e^{at}M_X(bt)$

d)If X and Y are independent and $Z=X+Y$ then $M_Z(t)=M_X(t)\,M_Y(t)$

Example 1) X~Uniform[0,1]

$M_X(t)=$

$M_{aX+b}(t)=$

Example 2) Sum of Negative Binomials.

$$M(t)=\frac{(pe^t)^r}{[1-(1-p)e^t]^r} \text{ for } t<-\ln(1-p)$$