STAT 702/J702

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-Lecture 20-

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Today

- Homework
- More on Expected Values
- Moment Generating Functions

Ch 3: #66) First find the c.d.f. of $X_{(k)}$ and then find the p.d.f. by differentiating.

Problem 2) Assume a sample of size 30 is supposed to have come from a standard normal distribution. What is the 99th-%ile for the 30th order statistic?

<u>Chapter 4 Revisited:</u>
<u>More on Expected Values</u>

For constants a and b,

$E(a + bX) = a + b E(x)$

$Var(a + bX) = b^2 Var(X)$

Let $X_1, X_2, \ldots X_n$ be mutually independent random variables, then:

$\mu_{\Sigma X} = E(\Sigma_i X_i) = \Sigma_i E(X_i) = \Sigma \mu_{Xi}$

$\sigma_{\Sigma X}^2 = Var(\Sigma_i X_i) = \Sigma_i Var(X_i) = \Sigma\, \sigma_{Xi}^2$

What if the X_i are not independent?

First, if the X_i have joint p.d.f $f(x_1,\ldots x_n)$ and $Y=g(x_1,\ldots x_n)$ then

$$E(Y)=\int\cdots\int g(x_1,\ldots x_n)f(x_1,\ldots x_n)dx_1\cdots dx_n$$

Provided the integral converges with $|g|$.

Now consider $Y=a+b\sum_{i=1}^{n}X_i$

and finding E(Y) and Var(Y).

Covariance

$Cov(X,Y)=E[(X-\mu_X)(Y-\mu_Y)]$

Correlation

$Cor(X,Y)=\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$

4.5 – Moment Generating Functions

The moment-generating function (mgf) of X is $M(t)$=E(e^{tX})

$$M(t) = \sum_x e^{tX} p(x)$$

$$M(t) = \int_{-\infty}^{\infty} e^{tX} f(x) dx$$

Why "moment generating" ?

Assume the mgf exists on some interval around 0…

$$M(t) = \int_{-\infty}^{\infty} e^{tX} f(x) dx$$

$$M'(t) = \frac{d}{dt} \int_{-\infty}^{\infty} e^{tX} f(x) dx$$

Other properties:

a) The m.g.f. uniquely determines the p.d.f.

b) If Y=$a+b$X then $M_Y(t)=e^{at}M_X(bt)$

c) If X and Y are independent and Z=X+Y then $M_Z(t)=M_X(t)\ M_Y(t)$