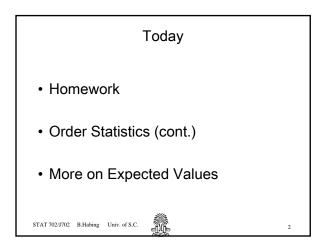
STAT 702/J702 October 26th, 2004 *-Lecture 19-*

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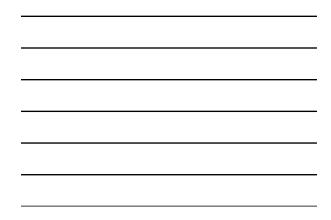
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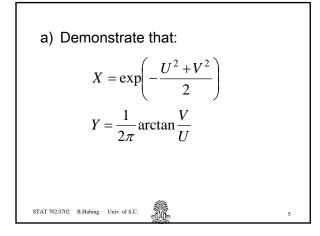


Ch 3: #38) Let T_1 and T_2 be independent exponentials with parameters λ_1 and λ_1 . Find the density function of T_1 and T_2 .

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Problem 2) Let X and Y be
independent uniform [0,1] random
variables.
Consider the (seemingly ugly)
transformations:
$$U = \sqrt{-2\ln(X)}\cos(2\pi Y)$$
$$V = \sqrt{-2\ln(X)}\sin(2\pi Y)$$





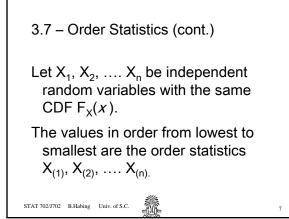


b) Use the transformation of variable formula to find the joint distribution of *U* and *V*, and remember to specify where it is defined.

c) Identify the joint distribution by name.

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The marginal p.d.f. for any of the order statistics is: $f_{X_{(k)}}(x_{(k)}) = \frac{n!}{(k-1)!(n-k!)} f_X(x_{(k)})$ $\cdot F^{k-1}(x_{(k)})[1-F_X(x_{(k)})]^{n-k}$ STAT 702/772 B.Habing Univ. of S.C.

The joint p.d.f. of all of the order statistics is: $f_{X_{(1)},...X_{(n)}}(x_{(1)},...x_{(n)}) =$ $n!f(x_{(1)})\cdots f(x_{(n)})$ One way to find the joint p.d.f. of a pair of order statistics would be to integrate out the *n*-2 you are not concerned with.

Another way is to use what the text calls "a differential argument" (Theorem A on 101 uses this to prove the result in the hmwk.)

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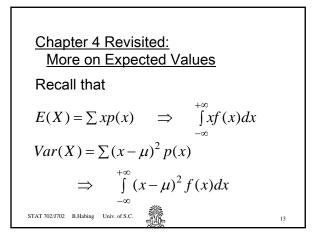
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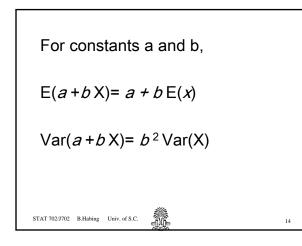
Say we want the joint p.d.f. of $X_{(i)}$ and $X_{(j)}$ where i<j. The trick to getting the joint p.d.f. directly is try to let our insights into discrete distributions apply to continuous random variables. In particular we will imagine that: $f(x, y) = P[x \le X \le x + dx, y \le Y \le y + dy]$ $f(x) = P[x \le X \le x + dx]$ STAT 702/J702 B.Habing Univ. of S.C. M

And so...
$$f_{X_{(i)},X_{(j)}}(x_{(i)},x_{(j)}) =$$

 $\cdot \frac{n!}{(i-1)!(j-i-1)!(n-j)!}$
 $\cdot F_X^{i-1}(x_{(i)})$
 $\cdot [F_X(x_{(j)}) - F_X(x_{(i)})]^{j-i-1}$
 $\cdot [1 - F_X(x_{(j)})]^{n-j}$
 $\cdot f_X(x_{(i)}) f_X(x_{(j)})$
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Let X₁, X₂, ... X_n be mutually independent random variables, then: $\mu_{\Sigma X} = E(\Sigma_i X_i) = \Sigma_i E(X_i) = \Sigma \mu_{Xi}$ $\sigma_{\Sigma X}^2 = Var(\Sigma_i X_i) = \Sigma_i Var(X_i) = \Sigma \sigma_{Xi}^2$

A

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What if the X_i are not independent?
First, if the X_i have joint p.d.f

$$f(x_1,...,x_n)$$
 and Y= $g(x_1,...,x_n)$ then
 $E(Y) = \int \cdots \int g(x_1,...,x_n) f(x_1,...,x_n) dx_1 \cdots dx_n$
Provided the integral converges with
 $|g|$.

