STAT 702/J702

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-*Lecture 19*-

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Today

- Homework
- Order Statistics (cont.)
- More on Expected Values

Ch 3: #38) Let T_1 and T_2 be independent exponentials with parameters λ_1 and λ_1. Find the density function of T_1 and T_2.

Problem 2) Let X and Y be independent uniform [0,1] random variables.

Consider the (seemingly ugly) transformations:

$$U = \sqrt{-2\ln(X)}\cos(2\pi Y)$$

$$V = \sqrt{-2\ln(X)}\sin(2\pi Y)$$

a) Demonstrate that:

$$X = \exp\left(-\frac{U^2+V^2}{2}\right)$$

$$Y = \frac{1}{2\pi}\arctan\frac{V}{U}$$

b) Use the transformation of variable formula to find the joint distribution of U and V, and remember to specify where it is defined.

c) Identify the joint distribution by name.

3.7 – Order Statistics (cont.)

Let X_1, X_2, …. X_n be independent random variables with the same CDF $F_X(x)$.

The values in order from lowest to smallest are the order statistics $X_{(1)}$, $X_{(2)}$, …. $X_{(n)}$.

The marginal p.d.f. for any of the order statistics is:

$$f_{X_{(k)}}(x_{(k)}) = \frac{n!}{(k-1)!(n-k!)} f_X(x_{(k)}) \cdot F^{k-1}(x_{(k)})[1 - F_X(x_{(k)})]^{n-k}$$

The joint p.d.f. of all of the order statistics is:

$$f_{X_{(1)},\ldots X_{(n)}}(x_{(1)},\ldots x_{(n)}) = n!\, f(x_{(1)}) \cdots f(x_{(n)})$$

One way to find the joint p.d.f. of a pair of order statistics would be to integrate out the *n*-2 you are not concerned with.

Another way is to use what the text calls "a differential argument" (Theorem A on 101 uses this to prove the result in the hmwk.)

Say we want the joint p.d.f. of $X_{(i)}$ and $X_{(j)}$ where i<j.

The trick to getting the joint p.d.f. directly is try to let our insights into discrete distributions apply to continuous random variables.

In particular we will imagine that:

$$f(x,y) = P[x \le X \le x+dx, y \le Y \le y+dy]$$

$$f(x) = P[x \le X \le x+dx]$$

And so…

$$f_{X_{(i)},X_{(j)}}(x_{(i)},x_{(j)}) = \cdot\frac{n!}{(i-1)!(j-i-1)!(n-j)!} \cdot F_X^{\,i-1}(x_{(i)}) \cdot [F_X(x_{(j)}) - F_X(x_{(i)})]^{j-i-1} \cdot [1-F_X(x_{(j)})]^{n-j} \cdot f_X(x_{(i)})\, f_X(x_{(j)})$$

Chapter 4 Revisited:
More on Expected Values

Recall that

$$E(X) = \sum xp(x) \quad \Rightarrow \quad \int_{-\infty}^{+\infty} xf(x)dx$$

$$Var(X) = \sum (x-\mu)^2 p(x)$$

$$\Rightarrow \quad \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)dx$$

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For constants a and b,

E(*a* + *b* X)= *a* + *b* E(*x*)

Var(*a* + *b* X)= b^2 Var(X)

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Let X_1, X_2, … X_n be mutually independent random variables, then:

$\mu_{\Sigma X}$ = $E(\Sigma_i X_i) = \Sigma_i E(X_i) = \Sigma \mu_{Xi}$

$\sigma_{\Sigma X}^2 = Var(\Sigma_i X_i) = \Sigma_i Var(X_i) = \Sigma\, \sigma_{Xi}^2$

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What if the X_i are not independent?

First, if the X_i have joint p.d.f $f(x_1,\ldots x_n)$ and Y=$g(x_1,\ldots x_n)$ then

$$E(Y) = \int \cdots \int g(x_1,\ldots x_n) f(x_1,\ldots x_n) dx_1 \cdots dx_n$$

Provided the integral converges with $|g|$.

Now consider $Y = a + b\sum_{i=1}^{n} X_i$

and finding E(Y) and Var(Y).