STAT 702/J702

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-Lecture 17-

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Today

- Homework Solution
- Functions of Joint Random Variables (continued)

Homework 11) Consider the joint pdf $f(x,y)=1+a(1-2x)(1-2y)$ defined for $0 < x,y < 1$, $-1 < a < 1$.

Find what conditions must be met for X and Y to be independent.

3.6.2 – Functions of Joint Random Variables

(X, Y) have joint pdf $f_{XY}(x, y)$.

We want the distribution of $U=g_1(X, Y)$, $V=g_2(X, Y)$.

E.g. : $U=X+Y$, $V=Y^2$ or $U=X+Y$, $V=X-2Y$, etc.

For the continuous case the joint pdf of (U,V) is

$$f_{U,V}(u, v) = f_{X,Y}(h_1(u, v), h_2(u, v))\,|J|$$

where h_1 and h_2 are the inverse functions: $x=h_1(u, v)$, $y=h_2(u, v)$)

And J is the Jacobian $J = \begin{vmatrix} \frac{dh_1}{du} & \frac{dh_1}{dv} \\ \frac{dh_2}{du} & \frac{dh_2}{dv} \end{vmatrix}$

Example 1) X and Y have joint p.d.f.
$f_{XY}(x,y) = 2 \quad 0 \le x < y \le 1$

U=X/Y and V=Y

Find the joint and marginal p.d.f.'s of X and Y.

Example 2) X and Y are bivariate normal with means 0, variances 1, and correlation = 0.

Let $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Find the joint and marginal distributions.

3.6.1 - Special Case 1 – Convolution

In general say Z=X+Y

We can find a general formula for $F_Z(z)$=P(Z<*z*) simply by finding the appropriate area under *f*(*x*,y).

Taking the derivative then gives us the pdf.

Example) X and Y are exponential RVs with parameter λ.

3.6.1 - Special Case 2 – Quotient

A general formula for the quotient $Z=Y/X$ can also be derived by examining the CDF.

To do this easily, note that if $y/x \leq z$ then if $x>0$ we have $y \leq xz$ and if $x<0$ then $y \geq xz$.

Back to the earlier example)

X and Y have joint p.d.f.

$f_{XY}(x,y) = 2 \quad 0 \leq x < y \leq 1$

$U=X/Y$

3.7 – Order Statistics

Let $X_1, X_2, \ldots X_n$ be independent random variables with the same CDF $F_X(x)$.

The values in order from lowest to smallest are the order statistics $X_{(1)}, X_{(2)}, \ldots X_{(n)}$.